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Geometric constructions of nongeometric string theories

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Abstract: We advocate a framework for constructing perturbative closed string compactifications which cannot be scaled up uniformly to have large-radius limits. The idea is to augment the class of vacua which can be described as fibrations by enlarging the monodromy group around the singular fibers to include perturbative stringy duality symmetries. As a controlled laboratory for testing this program, we study in detail six-dimensional (1,0) supersymmetric vacua arising from two-torus fibrations over a two-dimensional base. We also construct some examples of two-torus fibrations over four-dimensional bases, and comment on the extension to other fibrations.

Keywords: Superstrings and Heterotic Strings, String Duality, Superstring Vacua, Anomalies in Field and String Theories.

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Contents

1. Weakly coupled supersymmetric string vacua without geometry

In this paper we will examine a new method for constructing supersymmetric, nongeometric string theories. The examples on which we focus most closely make up a class of solutions to supergravity in $7 + 1$ dimensions with 32 supercharges. These solutions will involve nontrivial behavior of the metric and Neveu-Schwarz (NS) B-field, but not of any of the Ramond-Ramond fields, nor of the eight dimensional dilaton. (The ten-dimensional dilaton will vary, but only in such a way that the eight dimensional effective coupling is held fixed.) We will argue that these backgrounds are likely to represent sensible backgrounds for string propagation on which the dynamics of string worldsheets are determined by a two-dimensional conformal field theory of critical central charge, with a controlled genus expansion whose expansion parameter can be made arbitrarily small.

Almost all known examples of perturbative string backgrounds are descibed by *nonlinear sigma models*, that is, by field theories containing scalar fields X^μ parametrizing a topologically and geometrically nontrivial target space. The lagrangian for these theories,

$$L_{\text{worldsheet}} = \frac{1}{2\pi\alpha'} G_{\mu\nu}(X)\partial_+ X^\mu \partial_- X^\nu \,, \tag{1.1}$$

describes the dynamics of a fundamental string travelling in a curved spacetime with metric $G_{\mu\nu}$. The conditions for conformal invariance of the worldsheet theory are then

$$0 = \beta_{\mu\nu}[G_{\sigma\tau}(X)] = R_{\mu\nu}(X) + (\text{quadratic in } R_{\alpha\beta\gamma\delta}) + \cdots \tag{1.2}$$

for weak curvatures $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} \ll 1$ in string units. Therefore the condition for conformal invariance is approximately the same as the Einstein equation for the target space metric. So a nonlinear sigma model whose target space is a smooth Ricci-flat manifold at large volume will always be approximately conformal, an approximation which improves if one scales up the manifold $G_{\mu\nu} \to \Lambda G_{\mu\nu}$.

The existence of a large-volume limit of a family of solutions or approximate solutions gives rise to the *moduli problem*: there are usually several massless scalars in the lower-dimensional effective field theory corresponding to Ricci-flat deformations of the target space. There is always at least one light scalar, or approximate *modulus* — namely, the overall size of the compact space —- if flat ten-dimensional space is an exact solution to the string equations of motion at the quantum level. In critical superstring theory, to which we will restrict our attention exclusively in this paper, flat ten dimensional space *is* a solution to the equations of motion at the quantum level.

So the overall volume modulus is an intrinsic difficulty for geometric compactifications of superstring theory. Even if potentials are generated for the volume modulus, supersymmetry forces such potentials to vanish in the large volume limit, giving rise to potentials which attract the theory to its least phenomenologically interesting point [1].

It is therefore important to find compactifications of the theory which lack an overall scale modulus. We would like to do this at the perturbative level, by constructing backgrounds where the closed string worldsheet is controlled by a 2D conformal field theory which is not a sigma model, and which cannot be scaled up uniformly to give a large volume limit. To do this, one could in principle scan the space of two dimensional superconformal field theories of appropriate central charge, calculate their spectra, and consider only those not connected by marginal deformations to flat ten-dimensional space; in practice this would be prohibitively difficult.

Our strategy is to exploit the existence of stringy discrete gauge symmetries of partially compactified string theory, symmetries which do not commute with the operation $G_{\mu\nu} \to \Lambda G_{\mu\nu}$ of rescaling the metric. There are, indeed, three ways in which one could in principle exploit such symmetries in order to eliminate the overall volume modulus:

- I. One could orbifold by them, which would project out the volume modulus.

- II. One could simply consider points in the moduli space of compactifications under which all moduli, including the volume modulus, are charged; such a vacuum is guaranteed to be stationary (though not necessarily stable) against all quantum corrections.

- III. One could consider solutions to string theory with boundary conditions which lift the overall volume modulus.

The first possibility has been studied, in the form of the 'asymmetric orbifold' [2]–[5], which refers to an orbifold of a torus by a 'stringy' symmetry, such as T-duality, which has no classical counterpart and under which the volume transforms nontrivially. The second possibility has also been proposed, [6], as a mechanism for solving the moduli problem. In this paper we investigate the third possibility.[1]

[1]Some families of solutions of kind (III) will also contain solutions of kind (I), in the same way that ordinary Calabi-Yau manifolds often have orbifold limits at special points in moduli space.

JHEP01(2004)024

1.1 The setup

The idea is as follows: spacetime is a product of $10-n$ Minkowski dimensions with an internal space X_n. The internal space is a fiber product of a k-dimensional fiber space G over a $n-k$-dimensional base $\mathcal{B}$. Fiberwise, G solves the string equations of motion (meaning, in the cases we consider, that it is a flat T^2), and we denote by $\mathcal{M}$ the moduli space of k-dimensional solutions whose topology is that of G. We then allow the moduli $\mathcal{M}$ of the fiber to vary over $\mathcal{B}$ in such a way that the total space X_n also solves the equations of motion. Solving the equations of motion will usually force the lower-dimensional effective solution to break down at some singular locus $\mathcal{S}$. We emphasize that the microscopic solution generally does *not* break down at $\mathcal{S}$.

We focus on solutions in which, as one circumnavigates the singular locus $\mathcal{S}$ of codimension two in $\mathcal{B}$, the moduli $\mathcal{M}$ transform nontrivially under elements of the discrete symmetry group $\mathcal{G}$ acting on $\mathcal{M}$.

The adiabatic approximation in which G is fiberwise a solution is known in the geometric context as the semi-flat approximation ([7]). We will use these two terms interchangably. The semi-flat approximation typically breaks down in the neighborhood of $\mathcal{S}$. Nonetheless when the approximation breaks down, the resulting singularities can be understood, in all examples we consider, by using their local equivalence to other known solutions of string theory. An analogy which we have found useful is the following:

Dirac monopole **:** *'t Hooft-Polyakov monopole* **::** *semi-flat metric* **:** *smooth string background.*[2]

The Dirac monopole is a solution to a low-energy description (abelian gauge theory) of a microscopically well-defined field theory (for instance SU(2) gauge theory with a Higgs field in the adjoint). This solution has singular behavior near the core. For example, its inertial mass (the integrated hamiltonian density of the solution) is divergent due to an infinite contribution from the neighborhood of the origin. The 't Hooft-Polyakov monopole, a classical solution of the microscopic theory with the same charges as the Dirac monopole, is smooth at the core, but at distances up to the inverse Higgs vev behaves just like the Dirac solution. Had we not known how the abelian monopole field should behave near its core, we could learn this from the 't Hooft-Polyakov solution. This is our perspective on the semi-flat approximation – we use the correct microscopic physics in small patches when it becomes necessary, and use the semi-flat approximation to glue these patches together globally.

In all examples in this paper, the fiber G will be a torus, and we will focus on the case $G=T^2$ as a controlled laboratory for testing our ideas.

Keeping our fibration picture under strict control depends on the existence of a large-base limit. But this picture is a step towards building theories where even the large-base limit is absent. Further, these techniques allow for the exploration of a large class of

[2] In the case of purely geometric monodromies, the 'smooth string background' is literally a smooth ten-dimensional metric. In the case of nongeometric monodromies, there is no smooth metric but the solution is still demonstrably smooth as a string background. This can be shown using perturbative dualities and the known physics of NS fivebranes.

JHEP01(2004)024

heretofore unknown solutions of string theory. The new ingredients of our construction can be combined with other attempts at fixing moduli in ways that seem likely to be fruitful. We will briefly discuss this possibility in the concluding section.

The organization of the paper is as follows. In section 2, after a brief review of string theory on T^2, we will derive the effective equations for the metric and moduli in $\mathcal{B}$ which express the higher-dimensional equations of motion, specializing for concreteness to the case of two-dimensional base. We then analyze the allowed boundary conditions near the singular locus $\mathcal{S}$, and solve those equations locally in the case $\mathcal{B} = \mathbb{C}$ or $\mathbb{CP}^1$. We explain the six-dimensional effective dynamics of these theories involving the degrees of freedom localized at the degenerate fibers. In section 3 we will construct a global, compact solution. We deduce its spectrum, and check its consistency via anomaly cancellation in section 4. Section 5 presents some alternative descriptions of this background. In section 6 we consider the case where $\mathcal{B}$ is four-dimensional. In section 7 we discuss some the many directions for future study. In several appendices we collect some useful information about torus fibrations and type-IIA supergravity, and study a generalization of our ansatz.

2. Stringy cosmic fivebranes

In this section we will find and solve the conditions for supersymmetric solutions of type-IIA string theory which locally look like compactification on a flat T^2 fiber. We allow the moduli τ and ρ to vary over a two-dimensional base $\mathcal{B}_2$ and to reach degenerate values on some locus $\mathcal{S}$.

In bosonic string theory, a restricted class of such solutions where only τ *or* ρ varies has been known for some time [8]. We will try to make clear the connection to earlier work as we proceed, since some of our models have been made using other techniques. The idea of extending the monodromy group beyond the geometric one has been explored previously in e.g. [9, 10]. The emphasis in that work was on finding a higher-dimensional geometric description of the construction which we do not require. In addition, we have required that our models have a perturbative string theory description.

2.1 Type-IIA string theory on T^2

A few basic facts about type-II string theory on a flat two-torus will be useful to us. We will write the metric on the T^2 as

$$M_{IJ} = \frac{V}{\tau_2} \begin{pmatrix} |\,\tau\,|^2 & \tau_1 \\ \tau_1 & 1 \end{pmatrix}_{IJ} . \tag{2.1}$$

It is convenient to pair the moduli of the torus into the complex fields $\tau = \tau_1 + i\tau_2$ and $\rho = b + iV/2$ where

$$b \equiv \int_{T^2} B$$

is the period of the NS B-field over the torus. In eight-dimensional Einstein frame, the relevant part of the bosonic effective action for these variables is

$$S = \int_{M_8} d^8x \sqrt{g} \left(R + \frac{\partial_\mu \tau \partial^\mu \bar{\tau}}{\tau_2^2} + \frac{\partial_\mu \rho \partial^\mu \bar{\rho}}{\rho_2^2} \right) . \tag{2.2}$$

These kinetic terms derive from the metric on the moduli space which is invariant under the

$$\mathcal{G} \equiv O(2,2;\mathbb{Z}) \sim \mathrm{SL}(2,\mathbb{Z})_\tau \times \mathrm{SL}(2,\mathbb{Z})_\rho \tag{2.3}$$

perturbative duality group, some properties of which we will need to use. Under the decomposition indicated in (2.3), the first $\mathrm{SL}(2,\mathbb{Z})$ factor is the geometric modular group which identifies modular parameters defining equivalent tori. The second is generated by shifts of the B-field through the torus by its period:

$$b \mapsto b+1\,,$$

and by T-duality on both cycles combined with a 90° rotation of the T^2. We will generally use a prime to denote quantities associated with this second factor of the perturbative duality group.

One representation of this group comes from its action on windings and momenta of fundamental strings on the two-torus. Labelling these charges as w^I and p_I for $I=1,2$ along the two one-cycles, these transform in the $\mathbf{4} = (\mathbf{2},\mathbf{1}) \otimes (\mathbf{1},\mathbf{2})$ vector representation of this group. By this we mean that they transform as

$$\begin{pmatrix} p_I \\ \epsilon_{IJ} w^J \end{pmatrix} \mapsto \begin{pmatrix} a'\mathcal{T} & b'\mathcal{T} \\ c'\mathcal{T} & d'\mathcal{T} \end{pmatrix} \begin{pmatrix} p_I \\ \epsilon_{IJ} w^J \end{pmatrix} = \mathcal{T} \otimes \mathcal{T}' \begin{pmatrix} p_I \\ \epsilon_{IJ} w^J \end{pmatrix} \tag{2.4}$$

where $\mathcal{T} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $ad - bc = 1$.

Another representation of this group is on Ramond-Ramond (RR) charges. We can organize these into

$$\psi \equiv \begin{pmatrix} \text{D2 along } \theta_1 \\ \text{D2 along } \theta_2 \\ \text{D2 wrapped on } T^2 \\ \text{D0} \end{pmatrix} \tag{2.5}$$

which transforms in the reducible Dirac $(\mathbf{2},1) \oplus (1,\mathbf{2}) = \mathbf{2}_+ \oplus \mathbf{2}_-$ representation:

$$\psi \mapsto \begin{pmatrix} \mathcal{T} & 0 \\ 0 & \mathcal{T}' \end{pmatrix} \psi\,. \tag{2.6}$$

The periods

$$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \upsilon_1 \\ \upsilon_2 \end{pmatrix}$$

whose ratios are

$$\tau = \frac{\omega_1}{\omega_2} \qquad \rho = \frac{\upsilon_1}{\upsilon_2}$$

transform in this represenation.

Some further discrete symmetries will be relevant. We define $\mathcal{I}_2$ to be the transformation which inverts the torus:

$$\mathcal{I}_2 : \theta^I \mapsto -\theta^I\,.$$

In our notation above, this is $(\mathcal{T},\mathcal{T}') = (-1,1)$. $(-1)^{F_L}$ reverses the sign of all RR charges, and so can be written as $(\mathcal{T},\mathcal{T}') = (-1,-1)$. Note that this acts trivially on the vector (NS) representation as is to be expected.

2.2 Killing spinors

The action (2.2) was written in eight-dimensional Einstein frame, but it will be convenient to study the supersymmetry variations of the fermions in terms of string frame variables. We explain our index and coordinate and frame conventions in detail in the appendices. Actually, the results of this section apply to any G-fibration over $\mathcal{B}$, and we do not specify the dimension of the fiber or the base until the next subsection.

The supersymmetry transformations of the gravitino $\Psi_{\mu\alpha}$ and dilatino λ_α in type-IIA supergravity in string frame (see the appendices for details about frames and conventions) are

$$\delta\lambda = \left(\Gamma_{[10]}\Gamma^{\mu}\partial_\mu\Phi - \frac{1}{6}\Gamma^{\mu\nu\sigma}H_{\mu\nu\sigma}\right)\eta \tag{2.7}$$

$$\delta\Psi_\mu = \left(\partial_\mu + \frac{1}{4}\Omega_\mu{}^{\underline{\mathbf{MN}}}\Gamma_{\underline{\mathbf{MN}}}\right)\eta \tag{2.8}$$

where we have defined the generalized spin connection Ω as in [11] to be

$$\Omega_\mu^{\underline{\mathbf{MN}}} \equiv \omega_\mu{}^{\underline{\mathbf{MN}}} + H_\mu{}^{\underline{\mathbf{MN}}}\Gamma_{[10]}\,.$$

Here we have set to zero RR fields and fermion bilinears. η is a Majorana but not Weyl spinor of $\mathrm{SO}(9,1)$, and the NS field strength is

$$\begin{aligned} H_{\mu\nu\sigma} &\equiv (dB)_{\mu\nu\sigma} \equiv B_{\mu\nu,\sigma} + \text{cyclic} \\ H_{\mu\underline{\mathbf{MN}}} &\equiv H_{\mu\nu\sigma}E^{\nu}_{\underline{\mathbf{M}}}E^{\sigma}_{\underline{\mathbf{N}}}\,. \end{aligned} \tag{2.9}$$

The ten-dimensional chirality matrix is

$$\Gamma_{[10]} \equiv \frac{1}{10!}\epsilon_{\underline{\mathbf{M}}_1\cdots\underline{\mathbf{M}}_{10}}\Gamma^{\underline{\mathbf{M}}_1}\cdots\Gamma^{\underline{\mathbf{M}}_{10}} = \Gamma^{\dagger}_{[10]}\,. \tag{2.10}$$

We make an ansatz for the metric $G_{\mu\nu}$, NS two-form $B_{\mu\nu}$ and dilaton Φ of the form

$$\begin{bmatrix} G_{\tilde\mu\tilde\nu} & G_{\tilde\mu i} & G_{\tilde\mu J} \\ G_{i\tilde\nu} & G_{ij} & G_{iJ} \\ G_{I\tilde\nu} & G_{Ij} & G_{IJ} \end{bmatrix} = \begin{bmatrix} \exp\{w(x)\}\,\eta_{\mu\nu} & 0 & 0 \\ 0 & g_{ij}(x)+M_{KL}(x)A_i^K(x)A_j^L(x) & M_{JK}(x)A_i^K(x) \\ 0 & M_{IK}(x)A_j^K(x) & M_{IJ}(x) \end{bmatrix} \tag{2.11}$$

$$\begin{bmatrix} B_{\tilde\mu\tilde\nu} & B_{\tilde\mu i} & B_{\tilde\mu J} \\ B_{i\tilde\nu} & B_{ij} & B_{iJ} \\ B_{I\tilde\nu} & B_{Ij} & B_{IJ} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & B_{ij}(x) & B_{iJ}(x) \\ 0 & B_{Ij}(x) & B_{IJ}(x) \end{bmatrix} \tag{2.12}$$

$$\Phi = \Phi(x) \tag{2.13}$$

where all components of all fields depend only on the coordinates x^i on the base. This is the semi-flat approximation [8, 7]. It is the most general *ansatz* which preserves a $10-n$-dimensional Poincaré invariance and a $\mathrm{U}(1)^k$ isometry of the fiber.

For the moment we will further assume that

$$A_i^I(x) = B_{iI}(x) = 0\,, \tag{2.14}$$

which we will relax later on when studying the spectrum of fluctuations around our solutions. In the case where the monodromies are purely geometric, this additional restriction defines an *elliptic* fibration, in which the fiber is a holomorphic submanifold of the total space. In what follows we willuse the term elliptic more generally to describe the subset of compactifications where all of the the eight-dimensional vectors are turned off, even in the nongeometric case.

2.3 T^2 over $\mathcal{B}_2$

To our conserved spinor η we assign definite four-, six- and ten-dimensional chiralities

$$\Gamma_{[4]}\eta = \chi_4\eta \qquad \Gamma_{[6]}\eta = \chi_6\eta \qquad \Gamma_{[10]}\eta = \chi_{10}\eta \tag{2.15}$$

$$\Gamma_{[4]} \equiv \frac{1}{4!}\epsilon^{\underline{\mathbf{ABCD}}}\Gamma^{\underline{\mathbf{A}}}\cdots\Gamma^{\underline{\mathbf{D}}} = \Gamma^\dagger_{[4]} \tag{2.16}$$

$$\Gamma_{[6]} \equiv \Gamma_{[10]}\Gamma_{[4]} = \Gamma_{[4]}\Gamma_{[10]} \tag{2.17}$$

Next, we notice that the warp factor, $w(x)$, on the six-dimensional space vanishes given our ansatz.

The third simplifying observation is that gamma matrices in four dimensions satisfy the following identities:

$$\Gamma^{\underline{\mathbf{ABC}}} = \epsilon^{\underline{\mathbf{abcd}}}\Gamma_{[4]}\cdot\Gamma^{\underline{\mathbf{d}}} = -\epsilon^{\underline{\mathbf{ABCD}}}\Gamma^{\underline{\mathbf{D}}}\cdot\Gamma_{[4]} \tag{2.18}$$

$$\Gamma^{\underline{\mathbf{AB}}}\Gamma_{[4]} = -\frac{1}{2}\epsilon^{\underline{\mathbf{ABCD}}}\Gamma^{\underline{\mathbf{CD}}}. \tag{2.19}$$

The vanishing of the dilatino variation (2.7) is equivalent to

$$\chi_6\bar\partial\Phi = iV^{-1}\bar\partial b \tag{2.20}$$

and the vanishing of $\delta\Psi_I$ (where I is an index along the torus directions) is equivalent to

$$\Omega_I{}^{Aa} + \chi_4\epsilon^{AB}\epsilon^{ab}\Omega_I{}^{Bb} = 0\,. \tag{2.21}$$

We can write the generalized spin connection Ω whose (anti-)self-duality this equation expresses as

$$\Omega_I{}^{Aa} \equiv \omega_I{}^{Aa} + \chi_{10}\epsilon_{IJ}f^{AJ}e^{ai}b_{,i}\,. \tag{2.22}$$

In appendix D we show that this implies

$$\bar\partial\Phi = \bar\partial\ln\sqrt{V}\,, \tag{2.23}$$

which can be solved by setting

$$\frac{V}{e^{2\Phi}} \equiv g_8^{-2} \tag{2.24}$$

equal to a constant. We recognize this undetermined quantity g_8 as the eight-dimensional string coupling.[3] Given this relationship (2.24) between V and Φ, the remaining equations reduce to holomorphy of ρ and τ.

[3] Note that we would find other solutions for the dilaton if we turned on the RR potentials. In that case we would find F-theory-like solutions which could remove the zeromode of the dilaton as well as those of ρ and τ.

We show in appedix D.2 that the variation of the gravitino with index along the base will vanish if the conformal factor on the base satisfies

$$0 = \partial\bar{\partial}\left(\varphi - \ln\sqrt{\tau_2} - \ln\sqrt{\rho_2}\right). \tag{2.25}$$

2.3.1 Summary of Killing spinor conditions

We have found that if τ and ρ are holomorphic functions on the base, and the ρ degenerations back-react on the base metric in the same way as do the corresponding τ singularities [8], we preserve six-dimensional $(1,0)$ supersymmetry. With constant ρ we preserve $(1,1)$, with constant τ we preserve $(2,0)$. The complex structure on the base is correlated with the six-dimensional chirality.

2.4 Stringy cosmic fivebranes

The insight of [8] is that our moduli need only be single valued on the base up to large gauge identifications in the perturbative duality group $\mathcal{G}$. As such, there can be a collection $\mathcal{S}$ of branch points in $\mathcal{B}$ around which τ and ρ jump by some action of $g \in \mathcal{G}$. At such a degeneration point, the moduli must reach values fixed by the element g. Points in the moduli space fixed by elements of $\mathcal{G}$ will in general represent singular tori or decompactification points.

The basic example of a solution with nontrivial monodromy is

$$\tau = \frac{1}{2\pi i}\ln z\,.$$

In going around the origin, $z \mapsto e^{2\pi i}z$,

$$\tau \mapsto \tau + 1$$

which corresponds to the monodromy element

$$(\mathcal{T},\mathcal{T}') = \left(\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, 1\right)$$

in the notation of section 2.1.

A convenient way to encode the monodromy of τ is by describing the fiber tori as a family of elliptic curves satisfying a Weierstrass equation

$$y^2 = x^3 + f(z)x + g(z) \tag{2.26}$$

varying with z, a local coordinate on the base. For this elliptic fibration, the *discriminant locus* where the fiber degenerates is

$$\mathcal{S}_\tau = \{z \in \mathcal{B} \text{ s.t. } 0 = \Delta_\tau(z) = 4f(z)^3 + 27g(z)^2\}\,. \tag{2.27}$$

2.4.1 Local physics of the degenerations

In the previous subsections we have shown that we can preserve supersymmetry by letting ρ and τ vary as locally-holomorphic sections of a $\mathcal{G}$-bundle. Branch points around which the holonomy of the bundle acts represent degenerations of the T^2 fibration. The local physics of each possible degeneration is well-understood, and we review this understanding in this subsection.

Degenerations of the complex structure τ of an elliptic fibration were classified by Kodaira [12] according to the behavior of the polynomials in the Weierstrass equation (2.26). This classification (which appears as table 1 in the next section) tells us which kind of singularity of the total space of the fibration is created by a particular degeneration of the fiber. Assuming supersymmetry is preserved, these singularities have an ADE classification.

Note that an A_0 singularity is associated with the degeneration of a particular one-cycle of the T^2 fiber, which is some integer linear combination $p\mathbf{a} + q\mathbf{b}$ of the a-cycle and the b-cycle. Therefore, A_0 singularities come with a $\binom{p}{q}$ label.

2.4.2 Local physics of the nongeometric monodromies

What about degenerations of ρ? Consider a singular fiber near which

$$\rho \sim \frac{N}{2\pi i} \ln z \,. \tag{2.28}$$

Recalling that $\rho = b + iV/2$, this says that in going around the origin the B-field through the torus fiber moves through N periods,

$$b \mapsto b + N \,.$$

We can identify this object by performing a measurement of the H-flux through a surface surrounding the singularity. Such a surface is the T^2-fiber times a circle, C, around the origin, and $H = db \wedge \nu$ where ν is a unit-normalized volume-form on the T^2, so

$$\int_{T^2 \times C} H = \frac{N}{2\pi i} \oint_C \frac{dz}{z} = N \,. \tag{2.29}$$

This tells us that this homologically-trivial surface contains N units of NS5-brane charge [13]. Therefore, we identify a degeneration of the form (2.28) as the semi-flat description of a collection of N type-IIA NS5-branes. Note that this identification is consistent with the fact that a background with τ constant and ρ varying according to (2.28) preserves $(2, 0)$ supersymmetry, as does the IIA NS5-brane.

Next we must explain the microscopic origin of the other ρ degenerations, whose monodromies fill out the rest of $\mathrm{SL}(2, \mathbb{Z})_\rho$.

For this purpose it is useful to recall that T-duality along one cycle of the T^2 fiber replaces type-IIA with type-IIB. Choosing different one-cycles in the T^2 along which to dualize gives different IIB descriptions of a given IIA background. Start with the NS5-brane associated to the $\binom{1}{0}$ cycle (whose monodromy is

$$(\mathcal{T}, \mathcal{T}') = (1_{2\times 2}, (1 \quad 10 \quad 1)) \,.$$

JHEP01(2004)024

T-dualizing along the $\binom{1}{0}$ cycle, one obtains the type-IIB A_0 [13], and in particular the one associated with the $\binom{1}{0}$ cycle of the IIB torus. This is a KK monopole associated with momentum around this cycle. If we T-dualize back along the $\binom{p}{q}$ cycle of the type-IIB dual torus (i.e. transverse to the KK monopole), we obtain the object in type-IIA whose monodromy is

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \mapsto M_{p,q} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \tag{2.30}$$

where $\rho \equiv v_1/v_2$ and

$$M_{p,q} = \begin{pmatrix} 1-pq & p^2 \\ -q^2 & 1+pq \end{pmatrix}$$

is the $\mathrm{SL}(2,\mathbb{Z})$ matrix which preserves the vector $\binom{p}{q}$. It is important to notice that this monodromy includes some action on the volume of the T^2 for any (p,q) other than $(1,0)$.

To what extent is our semi-flat approximation valid near these objects? Clearly the actual ten-dimensional solution for an NS5-brane at a point on a T^2 will break translation invariance on the torus. In fact, evidence of this breaking is present even in the eight-dimensional smeared solution [14]. In particular, Kaluza-Klein momentum along the torus will not be conserved because of the H-flux. The important question to ask is about couplings. As we have said, the eight-dimensional string coupling is arbitrary. Both the volume of the fiber, and the ten-dimensional dilaton diverge near an NS5-brane. However tree-level string calculations not involving momentum modes on the torus are insensitive to this divergence.

2.5 Putting things on top of other things

Now that we have discussed the local physics near each kind of supersymmetric degeneration, we can consider configurations involving both types, which we know can preserve only one quarter of the original supersymmetry. Special examples of this type, those involving only A_N degenerations of ρ and arbitrary degenerations of τ, are locally described by collections of type-IIB NS5-branes probing ADE orbifold singularities (e.g. [15, 16, 17]), a configuration which is S-dual to D5-branes probing the orbifold as in [18]. However, if the configuration involves different kinds of degenerations (not just A_N) of both ρ and τ, it cannot be described in this way.

The metric on the base in the presence of both ρ and τ variation is given by $ds^2 = e^{2\varphi}|dz|^2$ where the conformal factor is

$$e^{2\varphi} = \tau_2 \left| \frac{\eta(\tau(z))^2}{\Pi_i (z-z_i)^{1/12}} \right|^2 \rho_2 \left| \frac{\eta(\rho(z))^2}{\Pi_i (z-\tilde{z}_i)^{1/12}} \right|^2$$

where η is the Dedekind eta function.

In the next section we construct and study in detail a compact model of this kind which preserves six-dimensional $(1,0)$ supersymmetry. As we will show, in a compact model in which both τ and ρ vary, one must have various degenerations of each type.

3. Compact models in six dimensions

What is required to make a compact model (whose base is a two-sphere with punctures) out of stringy cosmic fivebranes? There are two conditions one needs to satisfy. One is that the conformal factor on the base behaves smoothly at infinity, i.e. that the metric on the base is that of a sphere. The second condition is that the monodromy around all of the singularities be trivial, since this is the same as the monodromy around a smooth point in $\mathcal{B}$.

The first condition is easily satisfied by including the correct number of singularities, weigted appropriately. (The correct weighting of each singularity for this purpose counts the order of vanishing of the discriminant Δ of the corresponding elliptic curve [8].) Since the degenerations of ρ back-react on the metric on the base in the same way as do the degenerations of τ, we know from [8, 9] that this number is 24. This is because equation (2.25) tells us that φ is the two-dimensional electrostatic potential with charge equal to the tension E of the degenerations, which far from the degenerations behaves as $\varphi \sim -\frac{E}{2\pi} \ln |z|$. The total tension is $2\pi/12$ times the total number of times N that the maps

$$\tau : \mathbb{C} \to \mathcal{F}_\tau \,, \qquad \rho : \mathbb{C} \to \mathcal{F}_\rho$$

cover their fundamental domains, which says that far away the metric on the base looks like

$$ds^2 = e^{2\varphi} dz d\bar{z} \sim |z^{-N/12} dz|^2 \,;$$

with $N = 24$, the metric behaves as

$$ds^2 \sim \left| \frac{dz}{z^2} \right|^2$$

so that infinity is a smooth point in terms of $u = 1/z$, and we find $\mathcal{B} = \mathbb{P}^1$.

3.0.1 Trivial monodromy at infinity

The second condition can be solved in more than one way. Including the action on fermions and RR fields, it cannot be solved with fewer than 12 objects. For inspiration, we present table 1.

The column labeled "fiber type" gives Kodaira's name for the fiber type; the column labeled "singularity" gives the type of singularity the fiber degeneration causes in the total space. Note that, as we will clarify below, the contribution that the degeneration makes to the first chern class of the total space is $\mathrm{ord}(\Delta)/12$.

We have displayed this well-known classification of degenerations of an elliptic fibration to point out that the exceptional degenerations come in pairs (II^* and II, III^* and III, IV^* and IV, I_0^* and I_0^*) with monodromies inverse to each other. These pairs of degenerations all contain 12 simple singular fibers; a space with 12 simple degenerations is asymptotically cylindrical.

This suggests the following construction, which of course has more concrete descriptions which we give in subsequent sections. Choose any pair of τ degenerations which has trivial

$\mathrm{ord}(f)$	$\mathrm{ord}(g)$	$\mathrm{ord}(\Delta)$	monodromy	fiber type	singularity
≥ 0	≥ 0	0	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	smooth	none
0	0	n	$\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$	I_n	A_{n-1}
2	≥ 3	$n+6$	$\begin{pmatrix} -1 & -n \\ 0 & -1 \end{pmatrix}$	I_n^*	D_{n+4}
≥ 2	3	$n+6$	$\begin{pmatrix} -1 & -n \\ 0 & -1 \end{pmatrix}$	I_n^*	D_{n+4}
≥ 4	5	10	$\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$	II^*	E_8
≥ 1	1	2	$\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$	II	none
3	≥ 5	9	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	III^*	E_7
1	≥ 2	3	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	III	A_1
≥ 3	4	8	$\begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$	IV^*	E_6
≥ 2	2	4	$\begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$	IV	A_2

Table 1: Kodaira classification of singularities.

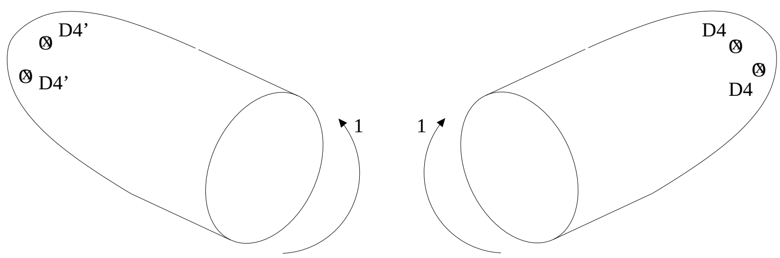

Figure 1: We can construct a compact model with both ρ and τ varying by gluing together two asymptotically cylindrical solutions, in each of which only one varies.

monodromy at infinity, and place them on a plane; this closes up the plane into a half-cigar. Then, independently choose a pair of ρ degenerations, also with trivial total monodromy, to make another half-cigar. The bases of these two two objects may then be glued along S^1 boundaries, as in figure 1, to make a compact space.

It is perhaps most convenient to consider the example depicted in figure 1 with two D_4 degenerations and two D_4' degenerations,[4] since in that case the values of ρ and τ can be constant and arbitrary.

[4]We denote by Γ' the degeneration of ρ which would be an orbifold singularity $\mathbb{C}^2/\Gamma$ if it were τ which were degenerating rather than ρ.

3.0.2 The $12 + 12'$ model

We can describe the model obtained by gluing these two cylinders by the following pair of Weierstrass equations:

$$y^2 = x^3 + f_4(z)x + g_6(z) \qquad \tilde{y}^2 = \tilde{x}^3 + \tilde{f}_4(z)\tilde{x} + \tilde{g}_6(z) \tag{3.1}$$

by defining

$$\tau(z) = j^{-1}\left(\frac{(12 f_4(z))^3}{4f_4(z)^3 + 27 g_6(z)^2}\right) \qquad \rho(z) = j^{-1}\left(\frac{(12\tilde{f}_4(z))^3}{4\tilde{f}_4(z)^3 + 27\tilde{g}_6(z)^2}\right) \tag{3.2}$$

where $j : UHP/\operatorname{SL}(2,\mathbb{Z}) \to \mathbb{C}$ is the elliptic modular function which maps the fundamental domain for the $\operatorname{SL}(2,\mathbb{Z})$ action on the upper half plane once onto the complex plane. We can think of the first equation of (3.1) as defining the complex structure of the actual torus fibers on which we have compactified type-IIA; the second equation determines its complexified Kähler form by specifying the complex structure of the mirror (T-dual along one cycle) torus. The degenerations of τ lie on the locus $\mathcal{S}_\tau$ of zeros of Δ_τ defined above in eq. (2.27) while the ρ-degenerations lie at the zero locus $\mathcal{S}_\rho$ of zeros of

$$\Delta_\rho(z) \equiv 4\tilde{f}_4(z)^3 + 27\tilde{g}_6(z)^2 \,.$$

The various points of enhanced symmetry described by the pairs of degenerations of inverse monodromy in table 1 can be reached by tuning the polynomials in (3.1) according to the table. We will, for convenience, count moduli at a generic point on this coulomb branch, where the zeros of Δ_τ and Δ_ρ are isolated.

4. The spectrum of the $12 + 12'$ model

4.1 "Elliptic" moduli

The coefficients of the polynomials f_4, g_6 and $\tilde{f}_4, \tilde{g}_4$ are moduli of our solution. f and g contain $5 + 7 = 12$ coefficients, of which a rescaling

$$f_4 \mapsto \lambda^2 f_4 \,, \qquad g_6 \mapsto \lambda^3 g_6$$

as in [9] does not change the torus, leaving 11 complex parameters. Similarly $\tilde{f}$ and $\tilde{g}$ give 11 complex parameters. Since these are all sections over a single $\mathbb{P}^1$, there is one overall $\operatorname{SL}(2,\mathbb{C})$ action on coordinates which removes three parameters leaving $22-3 = 19$ complex moduli of this kind.

4.2 RR vectors and tensors in six dimensions

In this part of the paper we wish to determine the light fields in six dimensions that arise from the ten dimensional RR forms. We will do this by reducing these forms along the fiber as sections of bundles over the base whose structure group is the monodromy group $O(2,2)$. We will then count the number of such sections in the semi-flat approximation. It will be useful throughout to keep in mind the example of K3 in the semi-flat approximation.

kind of 6d fields	where to put the indices	how many fields you get
scalars	A_1 on α^I	b^0_-
scalars	(A_1 on β^a, A_3 on $f \wedge \beta^a$)	b^1_+
scalars	A_3 on $b \wedge \alpha^I$	b^2_-
vectors	(A_1 on 1, A_3 on f)	b^0_+
vectors	A_3 on $\alpha^I \wedge \beta^a$	b^1_-
vectors	(A_3 on b, A_7 on $f \wedge b$)	b^2_+
tensors	A_3 on α^I	b^0_-
tensors	(A_5 on $f \wedge \beta^a$, A_3 on β^a)	b^1_+
tensors	A_5 on $b \wedge \alpha^I$	b^2_-

Table 2: Reduction of RR fields.

Reducing the Ramond-Ramond forms on the two-torus fiber, taking into account their transformation properties under the perturbative duality group $\mathcal{G}$, we learn that the number of RR tensors and RR vectors in six dimensions is

$$n_T = (b^0_- + b^1_+ + b^2_-) \qquad n_V = (b^0_+ + b^1_- + b^2_+)\,. \tag{4.1}$$

Here

$$b^p_\chi \equiv \dim_{\mathbb{R}} H^p\,(M, V_\chi)$$

and $H^p(M, V_\chi)$ is the pth cohomology with definite eigenvalue χ of the chirality operator on the Dirac representation of the $O(2,2;\mathbb{Z})$ bundle. Since from section 2.1 we know that the $O(2,2)$ chirality, χ, is simply inversion $\mathcal{I}_2$ of both directions of the torus, a form has a negative χ eigenvalue if it has an odd number of legs on the torus fiber. Since the Dirac representation is reducible, we may simply think of $V_\pm$ as independent rank two bundles.

In table 2, the objects $1, b, f, \alpha^I, \beta^a$ and wedges thereof are meant as place-holders to indicate where the indices of the RR potentials lie — respectively, all along the six dimensions, two along the base, two along the fiber, one along the fiber, one along the base.

We also find the same number n_T of real scalars which fit into the six-dimensional $(1,0)$ tensor multiplets. Note that 10-dimensional self-duality of RR fields relates the 6d scalars in the table to the same number of four-forms in six dimensions, and relates the 6d vectors to the same number of three-forms in six dimensions. However, it relates the tensors to themselves, rendering them self-dual, as is to be expected of the tensors in short multiplets of 6d $(1,0)$ supersymmetry.

$$n_T = 2\left(h^{(0,0)}_- + h^{(1,0)}_+\right), \qquad n_V = 2\left(h^{(0,0)}_+ + h^{(1,0)}_-\right),$$

where $h^{(p,q)}_\pm = \dim_{\mathbb{C}}\,(M, V_\pm)$.

4.3 Index theory on the base

From the previous subsection we know that in order to count the spectrum of light RR fields in the effective six-dimensional theory, we need to determine dimensions of some bundle-valued cohomology groups on the base.

JHEP01(2004)024

The first important point to make is that because the base is one complex dimensional, the number of sections is almost entirely determined by the abelian trace part of the bundle. The only effect of the non-abelian nature of the structure group is to reduce the number of fields by the rank of the representation except when the bundle is trivial.

We present the answer for the cohomology which we will justify in what follows. On the left we have included the results for K3.

For K3, one may infer these results from the fact that there are no RR tensors or scalars in the spectrum (which implies that $b^0_- = b^1_+ = 0$), and from the fact that there are 24 vectors. In this case the bundle V_+ is completely trivial since the monodromy group includes no action of $\mathcal{T}'$. A trivial rank 2 bundle on $\mathbb{P}^1$ has two scalar sections and no one-form sections. Therefore $b^0_+ = 2$. Note that this is consistent with index theory. The index theorem for the Dolbeault operator with values in the representation R of the bundle V says that [19]

$$\begin{aligned} h^{(0,0)}(\mathcal{B}, V) - h^{(1,0)}(\mathcal{B}, V) &\equiv \text{ind}\,(\partial_V) \\ &= \int_{\mathcal{B}} \text{Td}(\mathcal{B})\text{ch}_R(V) \\ &= \frac{1}{2}\dim(R)\chi(\mathcal{B}) + \frac{1}{2\pi}\int_{\mathcal{B}} \text{tr}\, F\,. \end{aligned} \tag{4.2}$$

Since the curvature F_+ of the bundle V_+ is trivial, dim $R = 2$ and $\chi(\mathcal{B}) = 2$, Applied to the Dolbeault operator with values in V_+ this gives:

$$h^{(0,0)}(\mathcal{B}, V_+) - h^{(1,0)}(\mathcal{B}, V_+) = 2\,. \tag{4.3}$$

Poincaré duality then tells us that $b^2_+ = 2$ as well and therefore, b^1_- must be 20 to account for the remaining vectors.

We will directly construct the relevant sections near the $D_4 \oplus D_4$ point, and then show that our counting is consistent with $O(2,2)$ index theory.

For definiteness let us place the singularities at the points $z = 0, 1, 2, 3$. In the case of K3, all four are D_4 singularities; in our $12 + 12'$ model, those at $z = 0, 1$ are D_4s, while those at $z = 2, 3$ are D'_4s. In each case, we place the branch cuts run between $z = 0, 1$ and between $z = 2, 3$. The metric on the base is the same in either case and is

$$ds^2 = \left| \frac{dz}{\sqrt{z(z-1)(z-2)(z-3)}} \right|^2 . \tag{4.4}$$

Note that infinity is a smooth point. We will count sections of V_- in each of the two cases; the symmetry between ρ and τ in the $12 + 12'$ model implies that the number of sections of V_+ is the same in that case.

At this point in the moduli space, it is easy to ignore the non-abelian structure, since the components do not mix under any of the visible monodromy elements. Note that this does not mean that they do not mix locally, close to the degenerations, and in fact we will assume that this is the case in performing our counting. So we may ignore the fact that the fundamental representation is actually a doublet, and, for K3, write a scalar in the **2** represenation of V_- as

$$\psi^{K3} = \frac{h_0^{K3}(z)}{\sqrt{z(z-1)(z-2)(z-3)}} \tag{4.5}$$

where the square roots in the denominator produce the correct monodromies, and $h_0^{K3}(z)$ is a single-valued function of z. A holomorphic 1-form valued section of this bundle is

$$\lambda^{K3} = \frac{h_1^{K3}(z)dz}{\sqrt{z(z-1)(z-2)(z-3)}} \tag{4.6}$$

where again h_1^{K3} is single-valued. In the $12 + 12'$ model, the scalar sections of V_- are

$$\psi = \frac{h_0(z)}{\sqrt{z(z-1)}} \tag{4.7}$$

and the holomorphic 1-form valued sections look like

$$\lambda = \frac{h_1(z)dz}{\sqrt{z(z-1)}} \tag{4.8}$$

Next we must demand that our sections are nonsingular at the smooth point $z = \infty$. For the scalar sections, replacing z with $u = 1/z$, the good coordinate at the other pole,

$$\psi^{K3}(u) = \frac{h_0^{K3}(1/u)u^2}{\sqrt{(1-u)(1-2u)(1-3u)}}\,.$$

So we see that the function $h_0^{K3}(z)$ can grow at most quadratically near infinity. For the one-form sections, $dz = -du/u^2$, so

$$\lambda^{K3}(u) = \frac{h_1^{K3}(1/u)du}{\sqrt{(1-u)(1-2u)(1-3u)}}$$

which implies that $h_1^{K3}(z)$ can only grow as a constant near infinity. In the $12 + 12'$ case we have

$$\psi(u) = \frac{h_0(1/u)u}{\sqrt{1-u}}$$

so h_0 can have at most a simple pole at infinity, while

$$\lambda(u) = \frac{h_1(1/u)du}{u\sqrt{1-u}}$$

so that h_1 must have a *zero* at infinity.

The functions $h_{0,1}$ also must behave properly near the singularities. At these singularities, the semi-flat approximation is badly wrong. However, as discussed above, the breakdown of the approximation only occurs in a small region around the singularity, and we may use our knowledge of the true *local* microscopic physics to determine conditions on the semi-flat fields at the boundary of this region.

JHEP01(2004)024

In this case, our knowledge of the K3 example tells us the desired information. Near a D_4 singularity, a scalar-valued section must have at least a simple zero, and a one-form-valued section can have at most a simple pole.[5] For K3, this tells us that $h_0^{K3}(z)$ is a single-valued function with four zeros that grows at most quadratically at infinity, and therefore vanishes. On the other hand, the function in the one-form sections on K3 are of the form

$$h_1^{K3}(z) = a + \frac{b_1}{z} + \frac{b_2}{z-1} + \frac{b_3}{z-2} + \frac{b_4}{z-3} \tag{4.9}$$

where the five numbers $a, b_1, \ldots, b_4$ are complex. There are therefore ten real $(1,0)$-form-valued sections of V_-, in agreement with table3.

This information that we have learned from K3 we can now apply to the $12+12'$ model. In this case, $h_0(z)$ must be a single valued function with two zeros in the complex plane which grows at most linearly at infinity, and so must vanish. The function in the one-form sections now *must* have a pole at one of the D_4 singularities since it must have a zero at infinity and so it is of the form

K3		$12+12'$	
$b^0_- = 0$	$b^1_- = 20$	$b^0_- = 0$	$b^1_- = 8$
$b^0_+ = 2$	$b^1_+ = 0$	$b^0_+ = 0$	$b^1_+ = 8$

Table 3: Cohomology in the semi-flat approximation.

$$h_1(z) = \frac{b_1}{z} + \frac{b_2}{z-1}\,. \tag{4.10}$$

Therefore, in the $12+12'$ model there are four independent real sections of this kind, and we learn that $b^1(\mathcal{B}, V_-) = 8$.

4.4 Scalars from NS vectors on the base

Another interesting component of the spectrum of this type of model arises from gauge fields on the base $\mathcal{B}$ which transform as sections in the vector representation of the $O(2,2;\mathbb{Z})$ bundle V. One-form-valued zeromodes of such fields (modulo gauge invariances) give rise to scalars in six dimensions. From the killing spinor conditions including the KK vectors and the B_{Ij} components, we know that this zeromode condition is simply that the connection be flat.

Therefore we can also reduce this component of the spectrum calculation to an index problem. The KK vectors and B-field vectors transform like string momenta and winding as in eq. (2.4). Therefore $\dim_{\mathbb{R}}(R) = 4$ and we should evaluate the first chern class term in (4.2) in the vector representation. The number on the left hand side of (4.2) is the number of scalar-valued sections in the vector representation minus the number of flat complex Wilson lines modulo gauge redundancies.

[5] Note that the naive norm on sections (the kinetic term for the resulting lower-dimensional fields in the semi-flat approximation) diverges logarithmically near the origin for a one-form-valued section when h_1 has a simple pole:

$$\|\lambda\|^2 = \int_{\Sigma_0} \lambda \wedge \bar{\lambda} = \int_{\Sigma_0} dz d\bar{z} \frac{|h_1(z)|^2}{|z(z-1)\cdots|} \sim \int_0 dr \frac{1}{r}$$

where Σ_0 is a small patch around the D_4 singularity at the origin. This is a breakdown of the semi-flat approximation exactly analogous to the divergence of the inertial mass of the Dirac monopole referred to in the introduction. We have circumvented this difficulty by inferring the local physics at the singularity from well-understood global facts about K3.

We may evaluate the chern class contribution to the index by picking a connection on V whose holonomy induces the desired monodromy on sections. Further, because we know the sections are loclized near the degenerations, we may evaluate the contribution of each singularity by examining only a small patch $\Sigma \subset \mathcal{B}$ around the singularity. Near a D_4 singularity at $z = 0$, such a connection, in the vector representation, is

$$A = A_z dz = \frac{q}{2i}\frac{dz}{z} 1_{4\times 4} \tag{4.11}$$

where q is an odd integer which is related to the allowed singular behavior of the sections near the D_4 point. The connection (4.11) has holonomy around a curve C surrounding the origin equal to

$$g_C = e^{i\oint_C A} = e^{i\pi q} = -1_{4\times 4}$$

as appropriate to a D_4 singularity.

The integer q can be determined by appealing to the results of the previous subsection, where we have learned that for K3

$$-10 = \mathrm{ind}(\partial_{V_-}) = 2 + 4c_1(D_4, \mathbf{2}) \tag{4.12}$$

where $c_1(D_4, \mathbf{2}) = \frac{1}{2\pi}\int_\Sigma \mathrm{Tr} F_-$ is the contribution to the first chern class of V_- from a region $\Sigma \subset \mathcal{B}$ containing *one* of the four D_4 singularities. Hence, $c_1(D_4, \mathbf{2})$ turns out to be -3. In this case the relevant connection is

$$A_- = A_z dz = \frac{q}{2i}\frac{dz}{z} 1_{2\times 2}$$

so we learn that

$$-3 = \frac{1}{2\pi}\int_\Sigma \mathrm{Tr} F_- = q\,.$$

In the vector representation, the connection appropriate to a D_4' singularity is the same, since the monodromies differ by a factor of $(-1)^{F_L}$ which does not act on the vector representation. Therefore, the contribution to the first chern class of *either* of these singularities is equal to

$$\frac{1}{2\pi}\int_\Sigma \mathrm{Tr}\, F = \frac{1}{2\pi}\oint_C \mathrm{Tr}\, A = 2q = -6 \tag{4.13}$$

where $C = \partial\Sigma$ is the boundary of the patch Σ containing the singularity at issue.

Therefore, the total index in the vector representation for the model with two D_4 singularities and two D_4' singularities is equal to

$$\mathrm{ind}\ (\partial_V) = \frac{1}{2}\cdot 4\cdot 2 - 4\cdot 6 = -20\,. \tag{4.14}$$

This says that zeromodes of vectors on the base contribute forty real scalars to the spectrum. Note that in the analogous calculation for type-II string theory on an elliptic K3, one also obtains forty real scalars in this manner, which combine with the 36 elliptic moduli plus four more real scalars from the complexified Kähler classes of the fiber and base to give the 80 real scalars in the 20 $(1,1)$ vectormultiplets.

4.5 Spectrum and anomaly

Putting together the results of this section, we find the following spectrum. In addition to the 8 tensor multiplets we found from RR fields, we find one more tensor multiplet which contains the $(1,3)$ piece of the unreduced NS B-field (the $(3,1)$ piece is in the six-dimensional $(1,0)$ gravity multiplet), and whose real scalar is the six-dimensional dilaton. We fnd two additional real scalars from the complexified Kähler class of the base, $\int_{\mathcal{B}}(B+ik)$. Therefore we have found $38 + 40 + 2$ real scalars in addition to the ones in the tensor multiplets. This says that we have at a generic point in the moduli space $n_T = 9$ tensor multiplets, $n_V = 8$ vector multiplets, and $n_H = 20$ hypermultiplets. These numbers satisfy the condition for the absence of an anomaly [20]:

$$n_H - n_V + 29n_T = 273\,, \tag{4.15}$$

a highly nontrivial check on the consistency of this background!

5. Other descriptions of these models

There is a simple sequence of fiberwise duality operations which relates our models to F theory models. Start with our theory in IIA with ρ and τ of a T^2 varying over $\mathcal{B}$. Make the IIA coupling finite so that, away from branch loci, we may describe it as M-theory on $S^1 \times T^2 = T^3$ over $\mathcal{B}$ and perform an 11-9 flip. The NS B-field ρ_1 along T^2 maps to the M-theory three-form along T^3. Further, note that because the ten-dimensional dilaton is not constant in our solutions, $R^{11} \propto \sqrt{\rho_2}$. If we reduce back to IIA along one of the one-cycles (say the θ^1-cycle) of the original T^2 we get a vacuum containing D6 branes at the degenerations of the torus, the C-field reduces to an NS B-field with its indices along 11 and the θ^2-cycle of the T^2. T-dualizing along the θ^2-cycle gives type-IIB with the axion-dilaton determined by τ; the D6 branes are replaced by D7 branes. This theory is compactified on a T^2 whose volume is determined by the original IIA coupling. Since the T-duality turns the NS B-field into shear along this torus, its complex structure is given by ρ. (The NS B-field through this torus arises from the wilson line of the RR 1-form along the θ^2-cycle of the original IIA solution, which lifts in M theory to shear between the original 11-direction and the θ^2-cycle). Clearly the information derived from our adiabatic duality can be incomplete when the semi-flat approximation breaks down. This incompleteness will be important when we try to compare tadpole cancellation in our models and their F-theory duals.

We end up with F-theory on the doubly-elliptically-fibered CY over $\mathcal{B}$ which defined as the complete intersection (3.1) in a $\mathbb{P}^2_{1,2,3} \times \mathbb{P}^2_{1,2,3}$-bundle over $\mathcal{B}$. In the case of the $12+12'$ model discussed in the previous section, this duality relates it to F theory on the doubly-elliptic Voisin-Borcea threefold labeled by $(r, a, \delta) = (10, 10, 0)$ and $(h^{1,1}, h^{2,1}) = (19, 19)$ which was found in [21] to have the above spectrum. Several other six-dimensional models with the same spectrum have been constructed [22, 23].

5.0.1 Branch structure

Models with six-dimensional $(1,0)$ supersymmetry can undergo transitions where the number of tensormultiplets decreases by one, and the number of hypermultiplets increases by 29. The prototypical example is the $E_8 \times E_8$ heterotic small-instanton transition [24, 25]. Via such a transition, described in the F theory dual in [21], it is possible for our model to reach a large-radius phase. These tensionless-string transitions descend to chirality-changing phase transition in four dimensions [26].

5.1 An asymmetric orbifold description[6]

As with many compactifications of string theory, including Calabi-Yau vacua, the $12 + 12'$ model has an exactly-solvable point in its moduli space. In particular, the following asymmetric orbifold of $T^3 \times R$ realizes a point in the moduli space of the $12 + 12'$ model:

$$\alpha : (\theta_1, \theta_2, \theta_3, x) \mapsto (-\theta_1, -\theta_2, -\theta_3, -x)$$

and

$$\beta : \begin{cases} (\theta_1, \theta_2, \theta_3, x) \mapsto (-\theta_1, -\theta_2, -\theta_3, L - x), \\ \Psi_\mu \mapsto \Gamma_{10} \Psi_\mu, \\ A^{RR}_{(p)} \mapsto -A^{RR}_{(p)} \end{cases} .$$

More succinctly, $\beta = (-1)^{F_L} \cdot \mathcal{P}_L \cdot \mathcal{I}_4$, where $\mathcal{P}_L$ is a translation by L in the x direction. The θs have period 2π. The fixed loci of α are (resolved) D_4 singularities at $(\theta_3, x) = (0, 0)$ and at $(\theta_3, x) = (\pi, 0)$. The fixed loci of β are (resolved) D_4' singularities at $(\theta_3, x) = (0, L/2)$ and at $(\theta_3, x) = (\pi, L/2)$. Note that the group element $\alpha \cdot \beta$ acts without fixed points.

Using the RNS formalism, one can compute the massless spectrum of this orbifold. The calculation is quite similar to that for the orbifold limit of K3, $T^4/\langle \mathcal{I}_4 \rangle$, the only differences arising from the reversal of the GSO projection in sectors twisted by $(-1)^{F_L}$. At each of the eight fixed loci of α, the massless states are the same as in the twisted sector of $\mathbb{R}^4/\langle \mathcal{I}_4 \rangle$, namely a vector from the twisted RR sector and the four real scalars of a $(1,0)$ hyper from the twisted NSNS sector. Each of eight fixed loci of β contribute the same twisted states as that of $\mathbb{R}^4/\langle \mathcal{I}_4 (-1)^{F_L} \rangle$. This consists of a self-dual tensor and a real scalar from the twisted RR sector, and the four real scalars of a $(1,0)$ hyper from the twisted NSNS sector. Unlike $T^4/\langle \mathcal{I}_4 \rangle$, the untwisted RR sector contributes nothing, while the untwisted NSNS sector is identical to the symmetric case — it contains 16 real scalars from $\psi^i_{-\frac{1}{2}} |0\rangle^{\rm untw}_{NS} \otimes \tilde{\psi}^j_{-\frac{1}{2}} |0\rangle^{\rm untw}_{NS}$ where i, j run over θ_i and x; this sector also contains the six-dimensional NSNS fields.

Thus we have found the matter content of the 6d $(1,0)$ gravity multiplet, 8 vectormultiplets, 8 tensormultiplets from RR fields plus one tensormultiplet containing the dilaton and half of the NS B-field, and 20 hypermultiplets. This is anomaly free and agrees with the spectrum computed in the previous section.

Therefore, one interpretation of our constructions, at least in this example, is as a useful description of deformations of asymmetric orbifolds away from the orbifold point.

[6]We are grateful to Mina Aganagic and Cumrun Vafa for discussions about such a description.

6. T^2 over $\mathcal{B}_4$

The most immediate way to extend these ideas beyond the six-dimensional $\mathcal{N} = (1,0)$ models is to enlarge the base to a space with four real dimensions. Generically, such solutions will preserve $\mathcal{N} = 1$ supersymmetry in four dimensions. In this section we comment on some preliminary studies of this interesting class of backgrounds. The feature on which we focus our comments is an eight-dimensional Chern-Simons term which represents a stringy correction to the action and BPS equations one obtains from naïve reduction of the ten-dimensional type-IIA action.

The BPS equations of the naïve 8D supergravity are satisfied if the base is a complex twofold with a Kähler metric $g_{i\bar{j}}$, the moduli τ and ρ both vary holomorphically over the base, and

$$\det g_{i\bar{j}} = \sqrt{\rho_2 \tau_2} \cdot f f^* \tag{6.1}$$

where $f(z_1, z_2)$ is a holomorphic function of the coordinates on the base.

By the argument given in section 5, these models are related to compactifications of F theory on doubly elliptic fourfolds. After an analysis of the local spectrum on the base and the introduction of some compact examples, we will comment further on this duality, and the manner in which the F theory tadpole [27, 28] is cancelled.

6.0.1 Local spectrum

Here we discuss the local physics at degenerations of the semiflat approximation, in type-IIA for concreteness. With a four-dimensional base, there are now three interesting classes of degenerate fibers. First, we can have a degeneration of τ in complex codimension one, which is now a complex curve in the base. In IIA string theory, an ADE degeneration of the fiber along a smooth curve of genus g gives ADE gauge symmetry with g adjoint hypers [29]. Secondly, ρ can degenerate in complex codimension one; an ADE NS5-brane wrapping a smooth genus g curve leads to $(\mathrm{ADE})^g$ gauge symmetry with one adjoint hyper, as e.g. in [30].

The new feature in the case when the base is a complex surface is that these degenerations can intersection Intersections of the either the τ discriminant or the ρ discriminant with itself are fairly well understood. Self-intersections of connected components of one or the other are included in the definition of 'genus' used above in stating the generic spectrum. Intersections of disconnected components give bifundamental matter [31] about which more below.

The intersection of the ρ discriminant and the τ discriminant is a new feature of this kind of model. Matter charged under the gauge group associated with a ρ singularity along $\mathcal{S}_\rho$ descends from objects which wrap around the one-cycles of $\mathcal{S}_\rho$ [30]. As a result, at a generic point in the moduli space (when these one-cycles are not pinched), we do not find massless matter localized at these $\rho - \tau$ intersections. Massive matter charged under both the ρ and τ groups, however, is another story, and will in general be present. Even in the case where we have a product of T^2 with the 4D space we constructed in section 3, there will be regimes of moduli space in which there are stable brane junctions charged under both groups.

To learn more about these new degenerations we construct some instructive compact examples.

6.0.2 Compact examples

A simple class of compact examples can be obtained by taking the base to be $\mathcal{B} = \mathbb{P}^1_1 \times \mathbb{P}^1_2$. Choose four asymptotically cylindrical collections of degenerations and place two at points on each of $\mathbb{P}^1_1$ and $\mathbb{P}^1_2$, wrapping the other $\mathbb{P}^1$. If we place all of the τ degenerations on $\mathbb{P}^1_1$ and all of the ρ degenerations on $\mathbb{P}^1_2$, we obtain, by the duality procedure of section 5, a model dual to F theory on $K3 \times K3$, which has enhanced supersymmetry.[7]

These examples fit into a more general collection in which the base is a Hirzebruch surface $\mathbb{F}_n$. We can define the models as in six dimensions by a Weierstrass equation for each of τ and ρ, (3.1) where the coefficients now depend on the coordinates of the $\mathbb{F}_n$ base. One way to be sure that we satisfy the global constraints discussed at the beginning of section 3 for a consistent compact model is to check that the dual F theory model is actually compactified on a compact Calabi-Yau fourfold. Explicitly, this fourfold is the complete intersection

$$y^2 = x^3 + f_4(stuv)xz^4 + g_6(stuv)z^6 \qquad \tilde{y}^2 = \tilde{x}^3 + \tilde{f}_4(stuv)\tilde{x}\tilde{z}^4 + \tilde{g}_6(stuv)\tilde{z}^6 \tag{6.2}$$

in the weighted $\mathbb{P}^2 \times \mathbb{P}^2$ bundle over $\mathbb{F}_n$ defined by the following toric data:

$$\begin{array}{ccccccccccc}
 & s & t & u & v & x & y & z & \tilde{x} & \tilde{y} & \tilde{z} \\
\lambda & 1 & 1 & n & 0 & 2\alpha & 3\alpha & 0 & 2\tilde{\alpha} & 3\tilde{\alpha} & 0 \\
\mu & 0 & 0 & 1 & 1 & 2\beta & 3\beta & 0 & 2\tilde{\beta} & 3\tilde{\beta} & 0 \\
\nu & 0 & 0 & 0 & 0 & 2 & 3 & 1 & 0 & 0 & 0 \\
\rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 3 & 1\,.
\end{array}$$

We need to specify a chamber for Kähler moduli in the quotient above. To obtain a positive volume Hirzebruch surface, we want to have $R^2_\lambda > nR^2_\mu > 0$. To have geometric torus fibers, we want $R^2_\nu, R^2_\rho > 0$. Further, the base is only a Hirzebruch surface for $n \leq 12$.

The conditions that (6.2) define a CY fourfold are

$$\alpha + \tilde{\alpha} = n + 2 \qquad \beta + \tilde{\beta} = 2\,. \tag{6.3}$$

We leave a more detailed study of these backgrounds for future work.

6.0.3 Tadpoles

F theory compactifications on CY fourfolds have tadpoles for the RR fourform potential polarized along the four noncompact spacetime directions [27, 28]. The magnitude of this tadpole, in units of D3-brane charge, is 1/24th of the Euler character of the fourfold. It may be cancelled by placing space-filling D3 branes at points on the base of the fourfold,

[7] An alternative dual description of this theory, in terms of the heterotic superstring on a non-Kähler space with H-flux, has been brought to our attention [32].

or by turning on crossed threeform fluxes on the six-dimensional base. The space-filling D3 branes can also be dissolved into the D7 branes as instantons.

The nongeometric string theories which we have described do not have such a tadpole. This may be seen, for example, from the fact that they have an orbifold limit, analogous to the one given for the $\mathcal{B}_2$ case in the previous section. This is a consistent (modular invariant) asymmetric orbifold, discussed in more detail below, which is connected to the theory by a smooth deformation of moduli during which the quantized value of the tadpole cannot change. This orbifold limit also makes it clear that $\mathcal{N} = 1$ supersymmetry is preserved.

As we have emphasized, the intersection of a ρ-singularity with a τ-singularity is a breakdown of the semi-flat approximation which is novel in the $\mathcal{B}_4$ case. The duality operation which takes us to F theory, since it uses T-duality on the T^2 fiber, relies on this approximation. As such, it may be misleading near points in the base where the approximation is wrong. Naive application of this duality tells us that a ρ-τ intersection maps to a gravitational instanton on a D7-brane worldvolume $\mathcal{S}$, which would carry (fractional) negative D3-brane charge, due to the coupling

$$- \int_{D7} C^{(4)}_{RR} \wedge \frac{p_1(\mathcal{S})}{24} \,,$$

which is one view of the origin of the tadpole [27]. We conjecture that the correct map actually gives such a 7-brane with a real D3-brane on top of the gravitational instanton, cancelling the tadpole globally. Note that such a D3-brane has a branched moduli space, which includes instanton moduli on the D7-brane, and coulomb branch moduli which move it in the two directions transverse to the D7-brane in the six-dimensional base of the elliptic fourfold [33]. The orbifold calculation below provides a check on this idea.

If we move one of these D3-branes to a point $p \in \mathcal{B}_4$, away from special loci ($\mathcal{S}_\tau$ and $\mathcal{S}_\rho$) in $\mathcal{B}_4$, we may trust our adiabatic duality map to determine its image in the non-geometric string theory. Reversing the duality chain of section 5, we first T-dualize along one of the one-cycles of the T^2 fiber of the threefold base. This turns the D3 brane into a D4 brane wrapping the θ^2-cycle of the T^2 fiber in the type-IIA background with D6 branes. Next, we lift to M theory, and interchange the θ^1-cycle with the M-direction. The D4-brane, which was not wrapping the θ^1-cycle, then becomes an NS5 brane wrapping the T^2 fiber over $p \in \mathcal{B}_4$. This then predicts that the intersection of a ρ degeneration and a τ degeneration can emit a wrapped NS5-brane. In the generic case the tadpole is cancelled globally but not locally (see for example [34] for a discussion of the F-theory dual to this effect), there will be a warp factor (as in (2.11)) and varying dilaton on the base, and the we leave the details of this for future work.

As further evidence for this picture, note that the Euler characteristic of a doubly elliptic fourfold can be rewritten as follows. Because $\chi(T^2) = 0$, we have (see e.g. [35] p. 148)

$$\chi(X) = \chi(\mathcal{S} \subset B_6)$$

where $\mathcal{S} = \{p \in B_6 \text{ s.t. } \Delta_\tau(p) = 0\}$ is the discriminant locus of the τ fibration. Because f, g are independent of the coordinates of the ρ torus $(\tilde{x}, \tilde{y}, \tilde{z})$, $\mathcal{S}$ wraps the ρ torus, and therefore is itself elliptic. Using the formula for the Euler character of an elliptic fibration once again,

$$\chi(X) = \chi(\mathcal{S}) = \chi(\mathcal{S}_\rho \cap \mathcal{S}_\tau \subset \mathcal{B}_4)$$

where $\mathcal{S}_\rho = \{p \in \mathcal{B}_4 \text{ s.t. } \Delta_\rho(p) = 0\}$ and $\mathcal{S}_\tau = \pi(\mathcal{S}) = \{p \in \mathcal{B}_4 \text{ s.t. } \Delta_\tau(p) = 0\}$ is the image in the four-dimensional base of the τ discriminant. But $\mathcal{S}_\rho \cap \mathcal{S}_\tau \subset \mathcal{B}_4$ is just a collection of points, so the tadpole (which we interpret as the number of D3 branes) is

$$\frac{\chi(X)}{24} = \frac{1}{24} \# \left(\mathcal{S}_\rho \cap \mathcal{S}_\tau\right)_{\mathcal{B}_4}.$$

This is nothing but the number of intersection points of the ρ-branes and the τ branes on the four-dimensional base, counted with multiplicity and divided by 24. This number can be measured by the integral formula

$$\mathcal{I} = C \int_{\mathcal{B}_4} \omega_\tau \wedge \omega_\rho = C \int_{\mathcal{B}_4} \frac{d\tau \wedge d\bar{\tau}}{-8i\tau_2^2} \wedge \frac{d\rho \wedge d\bar{\rho}}{-8i\rho_2^2}; \tag{6.4}$$

this integral counts the number of times the map

$$(\tau, \rho) : \mathcal{B}_4 \to \mathcal{F} \times \mathcal{F} \tag{6.5}$$

covers its image.[8]

6.0.4 Asymmetric orbifold limit

One technique we can use to analyze the spectrum, and to connect with known models, is to study an orbifold limit. For concreteness, we consider the following orbifold, which is in the moduli space of the model with two D_4s and two D_4's on each of two $\mathbb{P}^1$ factors of the base, shown in figure 2:

$$\frac{(T^2 \times S^1 \times \mathbb{R} \times S^1 \times \mathbb{R})}{\Gamma}$$

with coordinates $(\theta^8, \theta^9, \theta^4, x, \theta^6, y)$ (think of $\theta^{8,9}$ as parametrizing the T^2-fiber directions) where $\Gamma = \langle \alpha, \beta, \gamma, \delta \rangle$ and

$$\begin{aligned}
\alpha &: (\theta^8, \theta^9, \theta^4, x, \theta^6, y) \mapsto (-\theta^8, -\theta^9, -\theta^4, -x, \theta^6, y)\\
\beta &: (\theta^8, \theta^9, \theta^4, x, \theta^6, y) \mapsto (-\theta^8, -\theta^9, -\theta^4, L - x, \theta^6, y) \otimes (-1)^{F_L}\\
\gamma &: (\theta^8, \theta^9, \theta^4, x, \theta^6, y) \mapsto (-\theta^8, -\theta^9, \theta^4, x, -\theta^6, -y)\\
\delta &: (\theta^8, \theta^9, \theta^4, x, \theta^6, y) \mapsto (-\theta^8, -\theta^9, \theta^4, x, -\theta^6, L - y) \otimes (-1)^{F_L}
\end{aligned}$$

which is very similar to the orbifold in section 5.

Other sectors $\alpha\beta\gamma$ etc.. contain no massless states.

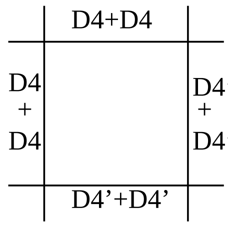

Figure 2: The most interesting of the examples with $\mathcal{B}_4 = \mathbb{P}^1_1 \times \mathbb{P}^1_2$ has a D_4 and a D_4' wrapping each projective line.

[8] To fix the normalization C, consider the example dual to $K3 \times K3$, where the map (6.5) factorizes. In this case, the Euler character of the fourfold is 24^2, which is also equal to the number of times $\mathcal{S}_\rho$ intersects $\mathcal{S}_\tau$. Therefore

$$(24)^2 = \mathcal{I}(K3 \times K3) = C \int_{\mathbb{P}^1_1} \omega_\tau \cdot \int_{\mathbb{P}^1_2} \omega_\rho = C \left(\frac{2\pi}{12}\right)^2$$

and $C = \left(\frac{24 \cdot 12}{2\pi}\right)^2$.

sector	massless fields	sector	massless fields
αRR	8 complex scalars	$\alpha NSNS$	8 complex scalars
βRR	8 vectors	$\beta NSNS$	nothing
$\alpha\gamma RR$	4 complex scalars	$\alpha\gamma NSNS$	4 complex scalars
$\alpha\delta RR$	4 vectors	$\alpha\delta NSNS$	nothing
γRR	8 complex scalars	$\gamma NSNS$	8 complex scalars
δRR	8 vectors	$\delta NSNS$	nothing
$\beta\delta RR$	4 complex scalars	$\beta\delta NSNS$	4 complex scalars
$\beta\gamma RR$	4 vectors	$\beta\gamma NSNS$	nothing

Table 4: Massless bosonic spectrum in the asymmetric orbifold limit.

One point to notice is that if one modifies γ, δ to something like

$$\gamma : (\theta^8, \theta^9, \theta^4, x, \theta^6, y) \mapsto (\epsilon - \theta^8, -\theta^9, \theta^4, x, -\theta^6, -y)$$
$$\delta : (\theta^8, \theta^9, \theta^4, x, \theta^6, y) \mapsto (\epsilon - \theta^8, -\theta^9, \theta^4, x, -\theta^6, L - y) \otimes (-1)^{F_L}$$

then $\alpha\delta$ no longer has any fixed locus, which means that at least in the orbifold limit, the spectrum of the $\alpha\delta$ twisted sector cannot be chiral, meaning that if it is ever chiral, the relevant gauge group is higgsed at the orbifold point.

The fixed locus of $\alpha\delta$ is the intersection of two $D_4 + D_4$s and two $D_4' + D_4'$s. Each D_4 or D_4' is made of 12 ρ or 12 τ branes respectively. This means that the dual tadpole is cancelled by $4 \times 6 \times 6/24 = 12$ D3 branes. There are 8 total $\rho - \tau$ intersection points in the base. Since 12 is not evenly divisible by 8, there must be some way for the tadpole-cancelling objects to fractionate. This situation is reminiscent of that of [36].

6.0.5 Chiral matter

We can use the duality with F theory to investigate chiral matter in these backgrounds. According to [31], matter in chiral representations of the geometric gauge group in F theory on a fourfold arises along curves $\mathcal{C}$ of intersection of components of the discriminant. Thinking of the components of the discriminant as collections of D7-branes in a IIB description, the $7-7'$ and $7'-7$ strings are bifundamentals. Twisting allows the $7-7'$ strings to have a different number of zero modes than the $7'-7$ strings. This difference is [31]

$$\chi_{\tau\tau} = 1 - g(\mathcal{S}) + c_1(\mathcal{L}) \tag{6.6}$$

where $\mathcal{L}$ is a line bundle over the intersection $\mathcal{S}$ determined by the topological twist on the 7-brane worldvolumes. This spectral asymmetry is just that arising from branes at angles [37] and as such $c_1(\mathcal{L})$ is related to the angle of intersection of the two components.

In our models this corresponds to matter charged under the gauge groups associated to different components of the τ discriminant locus. In the F theory dual, because f, g are independent of $\tilde{x}, \tilde{y}, \tilde{z}$, the curve $\mathcal{C}$ along which they intersect will always be the ρ-torus. Thus the chirality under the τ gauge group is simply $\chi_{\tau\tau} = c_1(\mathcal{L})$ in these examples. It seems likely that there is also interesting matter charged under the ρ gauge group at a self-intersection of the ρ-discriminant $\mathcal{S}_\rho$.

Note that it is not simply any self-intersection of $\mathcal{S}_\tau$ that leads to chiral matter. For example, in the model with $\mathcal{B}_4 = \mathbb{P}^1_1 \times \mathbb{P}^1_2$ and two geometric D_4's on each $\mathbb{P}^1$ factor, there is enhanced supersymmetry; the dual F theory model has a base which is the product $B_6 = T^2 \times \mathbb{P}^1_1 \times \mathbb{P}^1_2$. Clearly the components of the discriminant along $\mathbb{P}^1_1$ and those along $\mathbb{P}^1_2$ intersect along $\mathcal{C} = T^2$, the torus factor of B_6. However, because the branes intersect at right angles, there is no chiral matter in this case.

7. Discussion

7.0.6 Fixing the size of the base

The reader will have noticed that the moduli of the geometric fibration which are eliminated by the extension of the monodromy group are the moduli of the *fiber*. We have said nothing thus far about fixing the (complexified) kähler moduli of the base. However, for this purpose we may employ another mechanism for fixing moduli, which is also generically active in type-II string solutions. This is the presence of RR fluxes (see e.g. [38, 39, 40, 34, 41, 42]). This mechanism is actually quite complementary to our own.[9] One finds it difficult [34] to fix the overall size of the manifold using fluxes, whereas, as we have emphasized, nongeometric monodromies generically eliminate this mode.

7.0.7 T^3 fibrations and junctions of degenerate loci

From [7] we believe that a generic CY threefold is fibered by special lagrangian three-tori. This fibration degenerates along a real codimension two locus $\mathcal{S}$ in the base $\mathcal{B}$. Such a locus is a collection of curve segments joined at junctions [43].

In the same way that in the case of T^2 fiber we may think of the back-reaction of the discriminant locus as closing up the base into a compact manifold, we should think of $\mathcal{S}$ of the SYZ fibration as a *gravitating cosmic string junction* — or more precisely, a cosmic fourbrane junction, when the noncompact directions are included.

Along each segment, a particular $T^2 \subset T^3$ degenerates, and the monodromy around this component lies in a $\mathrm{SL}(2,\mathbb{Z}) \subset \mathrm{SL}(3,\mathbb{Z})$. In a geometric CY, the total monodromy around all components of $\mathcal{S}$ is a subgroup of the geometric $\mathrm{SL}(3,\mathbb{Z})$ modular group of the T^3. More generic solutions will extend this monodromy group to a larger subgroup of the duality group, and will preserve only four supercharges in four dimensions. Some early work in this direction includes [44].

7.0.8 Worldsheet CFT

As we have emphasized, since the eight dimensional string coupling is arbitrary, these models have weak-coupling limits in which many quantities are computable using worldsheet conformal field theory techniques. Away from orbifold limits, we have not addressed the interesting question of which CFT one should use. One point to notice is that the static gauge CFT on a long fundamental string has only $(0,4)$ worldsheet supersymmetry.[10] The

[9] We are grateful to Shamit Kachru for discussions on this point.

[10] This was emphasized to us by Mina Aganagic.

JHEP01(2004)024

worldsheet superalgebra of the RNS superstring prior to gauge fixing and GSO projection must be $(1,2)$ or $(1,4)$ in order to allow for a consistent set of superconformal constraints and reduced spacetime supersymmetry. We remind the reader that the worldsheet SUSY before gauge fixing and GSO projection and the static gauge SUSY of the long string need not be the same. Although the two superalgebras are the same in the case of CY3 compactification — namely, they are both $(2,2)$ — there is no natural identification between these algebras, one being (partly) a gauge artifact and the other being a physical symmetry which acts on the spectrum. We should not be surprised that the two superalgebras differ in our case.

7.0.9 Generalizations

There are two places during the course of our analysis where our ansatz was not the most general possibility.

One is in appendix E, where we noticed that it may be possible to preserve supersymmetry while turning on KK flux and H-flux in a specific linear combination. Such a possibility seems to be related to the mirror image of NS flux. As a clear missing piece in our understanding of possible type-II flux backgrounds, this is a subject of great current interest [45].

Secondly, we have turned off the RR potentials in our ansatz. We found in this ansatz that the volume of the base and the ten-dimensional IIA dilaton were related by (2.23). Turning on the RR potentials allows the dilaton to be a constant plus the imaginary part of a holomorphic function σ whose real part is an RR axion. Another $\mathrm{SL}(2,\mathbb{Z})$ which acts by fractional linear transformations on σ is obtained by conjugating $\mathrm{SL}(2,\mathbb{Z})_\rho$ by an 11-9 flip. Solutions where the monodromy group includes action by this $\mathrm{SL}(2,\mathbb{Z})$ will fix the dilaton, though not at weak coupling.[11] By the duality operation described in section 5, the type-IIA dilaton maps to the volume of the elliptic fiber of the F-theory base, which means that $\sigma \mapsto -1/\sigma$ jumps map to T-dualities in the geometry of the F theory base. Therefore a solution in which σ jumps maps to a non-geometric compactification of F-theory!

7.0.10 Extensions

We hope we have convinced the reader that the semi-flat approximation is a useful physical tool. When combined with knowledge about the microscopic origins of its *localized* breakdowns, it provides an effective handle on otherwise uncharted solutions. There is clearly an enormous class of string vacua which may be described this way; the elliptic fibrations on which we have focused are only a special case. Given a string compactification (not necessarily a geometric one) whose lower-dimensional effective theory, duality group, and extended objects are known, one can describe a large class of less-supersymmetric solutions with some of the moduli eliminated using a a nontrivial fibration of this theory whose monodromies lie in the full duality group of string theory on the fiber.

Partially geometric constructions of intrinsically stringy vacua make it possible to disentangle the properties of these vacua from the obscure machinery of worldsheet conformal

[11] An attempt to fix the dilaton by orbifolding by such a symmetry was made in [46].

field theory and combine them with such modern tools as Ramond-Ramond fluxes, branes, and F-theory. The prospect of lifting moduli with such a combination offers an inviting direction for future study.

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A. Conventions about coordinates and indices

We choose the following conventions for our coordinates: X^μ are the coordinates on the entire $9+1$ dimensions, $\tilde{y}^{\tilde{\mu}}$ are the coordinates on the $10-n$ Minkowski directions, x^i are the coordinates on the $n-k$-dimensional base, and θ^I are the coordinates on the T^k fiber.

Let $\underline{\mathbf{M}}, \underline{\mathbf{N}}, \underline{\mathbf{P}}, \cdots$ be the entire set of tangent space indices; let $a, b, c, \cdots$ be the tangent space indices corresponding to x^i; let $A, B, C, \cdots$ be the tangent space indices corresponding to θ^I; let $\underline{\mathbf{A}}, \underline{\mathbf{B}}, \underline{\mathbf{C}}, \cdots$ be indices that run over both a and A. We take the θ^I coordinates to have constant periodicity $\theta^I \sim \theta^I + n^I, n^I \in \mathbb{Z}$.[12]

B. Conventions for type-IIA supergravity

In this paper we use ten-dimensional string frame as the starting point for our conventions. Here we relate it to [47]. Before doing so we restore units and correct some minor errors in the published version of [47].

B.1 Clarifications and Corrections to [47]

- The Einstein term on page 327 of [47] should be multiplied by $1/k^2$.
- The definition of σ, on page 328, should read $\sigma \equiv \exp\{k\phi/(2\sqrt{2})\} = \exp\{\sqrt{2}k\phi/4\}$.
- Also note that the sign convention of [47] for defining the Ricci scalar is opposite that of the usual one, in which

$$[\nabla_\mu, \nabla_\nu] V^\sigma = R_{\mu\nu}{}^\sigma{}_\tau V^\tau \,. \tag{B.1}$$

[12]Yes, the period is 1 rather than 2π even though these coordinates are 'angles' denoted by 'θ^I'. Sorry!

B.2 Dictionary

Here we translate between the quantities of [47], which we denote with a [GP], and our quantities, which in this section we denote with an [US].

Define

$$k^{[\mathrm{GP}]} \equiv \frac{\sqrt{2}\alpha'^2}{2} \tag{B.2}$$

$$\phi^{[\mathrm{GP}]} \equiv \frac{1}{\alpha'^2}\Phi^{[\mathrm{US}]} \tag{B.3}$$

$$\Gamma_{11}^{[\mathrm{GP}]} \equiv \Gamma_{[10]}^{[\mathrm{US}]} \tag{B.4}$$

$$\sigma^{[\mathrm{GP}]} \equiv \exp\left\{\frac{\Phi^{[\mathrm{US}]}}{4}\right\} \tag{B.5}$$

$$G_{\mu\nu}^{[\mathrm{GP}]} \equiv \exp\left\{-\frac{\Phi^{[\mathrm{US}]}}{2}\right\} G_{\mu\nu}^{[\mathrm{US}]} \tag{B.6}$$

$$e_\mu^{\underline{\mathbf{A}}[\mathrm{GP}]} \equiv \exp\left\{-\frac{\Phi^{[\mathrm{US}]}}{4}\right\} E_\mu^{\underline{\mathbf{A}}[\mathrm{US}]} \tag{B.7}$$

$$A_{\mu\nu}^{[\mathrm{GP}]} \equiv -\frac{2}{\alpha'^2} B_{\mu\nu}^{[\mathrm{US}]} \tag{B.8}$$

$$\psi_\mu^{[\mathrm{US}]} \equiv \frac{\sqrt{2}\alpha'^2}{2} \exp\left\{\frac{\alpha'^2\phi^{[\mathrm{GP}]}}{8}\right\}\left(\psi_\mu^{[\mathrm{GP}]} - \frac{\sqrt{2}}{4}\Gamma_{[10]}\Gamma_\mu^{[\mathrm{GP}]}\lambda^{[\mathrm{GP}]}\right) \tag{B.9}$$

$$\lambda^{[\mathrm{US}]} \equiv 2\alpha'^2 \exp\left\{-\frac{\alpha'^2\phi^{[\mathrm{GP}]}}{8}\right\}\lambda^{[\mathrm{GP}]} \tag{B.10}$$

$$\epsilon^{[\mathrm{GP}]} = \exp\left\{-\frac{\Phi^{[\mathrm{US}]}}{8}\right\}\epsilon^{[\mathrm{US}]}. \tag{B.11}$$

Upon truncation to $N = 1$ in ten dimensions with a supersymmetry parameter of negative chirality, our conventions are identical to those of [11].

We now remind the reader of the transformations of various geometric quantities under a Weyl transformation $G_{\mu\nu}^{[\mathrm{OLD}]} = \exp\{2c\}G_{\mu\nu}^{[\mathrm{NEW}]}$:

$$E_\mu^{\underline{\mathbf{A}}[\mathrm{OLD}]} = \exp\{c\}\, E_\mu^{\underline{\mathbf{A}}[\mathrm{NEW}]} \tag{B.12}$$

$$E^{\underline{\mathbf{A}}\mu[\mathrm{OLD}]} = \exp\{-c\}\, E^{\underline{\mathbf{A}}\mu[\mathrm{NEW}]} \tag{B.13}$$

$$\Gamma_{\mu\nu}^{[\mathrm{OLD}]\sigma} = \Gamma_{\mu\nu}^{[\mathrm{NEW}]\sigma} + \delta_\mu^\sigma c_{,\nu} + \delta_\nu^\sigma c_{,\mu} - G_{\mu\nu}^{[\mathrm{NEW}]}G^{\sigma\tau[\mathrm{NEW}]}c_{,\tau} \tag{B.14}$$

$$\omega_\mu^{\underline{\mathbf{AB}}[\mathrm{OLD}]} = \omega_\mu^{\underline{\mathbf{AB}}[\mathrm{OLD}]} + c_{,\nu}\left[E_\mu^{\underline{\mathbf{A}}[\mathrm{NEW}]}E^{\nu\underline{\mathbf{B}}[\mathrm{NEW}]} - (\underline{\mathbf{A}} \leftrightarrow \underline{\mathbf{B}})\right] \tag{B.15}$$

$$R^{[\mathrm{OLD}]} = \exp\{-2c\}\left[R - (D-1)(D-2)(\nabla c)^2 - 2(D-1)(\nabla^2 c)\right]^{[\mathrm{NEW}]}. \tag{B.16}$$

With these definitions and identities, the IIA action translates from [47] as follows:

$$L^{[\mathrm{GP}]} = -\frac{e^{[\mathrm{GP}]}}{2k^{2[\mathrm{GP}]}}R^{[\mathrm{GP}]} - \frac{e^{[\mathrm{GP}]}}{2}(\nabla\phi)^{2[\mathrm{GP}]} - \frac{e^{[\mathrm{GP}]}}{12}\sigma^{-4[\mathrm{GP}]}(F_{\mu\nu\sigma}F^{\mu\nu\sigma})^{[\mathrm{GP}]} \tag{B.17}$$

$$= \frac{1}{\alpha'^4}\sqrt{-\det\, G^{[\mathrm{US}]}}\left(R^{[\mathrm{US}]} + 4(\nabla\phi)^{2[\mathrm{US}]} - \frac{1}{3}(H_{\mu\nu\sigma}H^{\mu\nu\sigma})^{[\mathrm{US}]}\right) \tag{B.18}$$

$$= L^{[\mathrm{US}]} \tag{B.19}$$

where we have dropped fermions, RR fields, and total derivatives. This bosonic NS action agrees with the action in [11], minus the terms from the gauge sector of the heterotic theory.

The supersymmetry variations of the fermions (again, setting to zero fermion multilinears and RR fields) are:

$$\delta\psi_\mu^{[\mathrm{US}]} = \nabla_\mu^{[\mathrm{US}]}\epsilon^{[\mathrm{US}]} + \frac{1}{4}(\delta_\mu^{[\lambda_1}\Gamma^{\lambda_2\lambda_3][\mathrm{US}]})\Gamma_{[10]}\epsilon^{[\mathrm{US}]}H^{[\mathrm{US}]}_{\lambda_1\lambda_2\lambda_3} \tag{B.20}$$

$$\delta\lambda^{[\mathrm{US}]} = -\Gamma^{\rho[\mathrm{US}]}\Gamma_{[10]}\epsilon^{[\mathrm{US}]}(\Phi^{[\mathrm{US}]}_{,\rho}) - \frac{1}{6}\Gamma^{\lambda_1\lambda_2\lambda_3[\mathrm{US}]}\epsilon^{[\mathrm{US}]}H^{[\mathrm{US}]}_{\lambda_1\lambda_2\lambda_3}\,. \tag{B.21}$$

For a gravitino of negative chirality this agrees, as noted before, with the supersymmetry transformations of [11].

C. Geometry of T^2 fibrations

Now we compute Christoffel symbols given our *ansatz* (2.11), (2.12), (2.14):

$$\Gamma_{\mu\nu;\sigma} \equiv \frac{1}{2}(G_{\mu\sigma,\nu} + G_{\nu\sigma,\mu} - G_{\mu\nu,\sigma}) \tag{C.1}$$

$$\Gamma_{ij;k} = \Gamma_{ij;k}{}^{[\mathcal{B}]} \qquad \Gamma^i_{jk} = \Gamma^i_{jk}{}^{[\mathcal{B}]} \tag{C.2}$$

$$\Gamma_{ij;I} = 0 \qquad \Gamma^I_{jk} = 0 \tag{C.3}$$

$$\Gamma_{iI;j} = 0 \qquad \Gamma^i_{jI} = 0 \tag{C.4}$$

$$\Gamma_{iI;J} = \frac{1}{2}M_{IJ,i} \qquad \Gamma^I_{iJ} = \frac{1}{2}M^{IK}M_{KJ,i} = \frac{1}{2}M^{IK}\nabla^{[\mathcal{B}]}{}_i M_{KJ} \tag{C.5}$$

$$\Gamma_{IJ;i} = -\frac{1}{2}M_{IJ,i} \qquad \Gamma^i_{IJ} = -\frac{1}{2}g^{ij}M_{IJ,j} = -\frac{1}{2}\nabla^{[\mathcal{B}]^i}M_{IJ} \tag{C.6}$$

$$\Gamma_{IJ;K} = 0 \qquad \Gamma^I_{JK} = 0 \tag{C.7}$$

$$R_{\mu\nu}{}^\sigma{}_\tau \equiv \Gamma^\sigma_{\nu\tau,\mu} + \Gamma^\sigma_{\mu\alpha}\Gamma^\alpha_{\nu\tau} - \Gamma^\sigma_{\mu\tau,\nu} - \Gamma^\sigma_{\nu\alpha}\Gamma^\alpha_{\mu\tau}\,, \tag{C.8}$$

defined in such a way that

$$[\nabla_\mu, \nabla_\nu]V^\sigma = R_{\mu\nu}{}^\sigma{}_\tau V^\tau\,, \tag{C.9}$$

where

$$\nabla_\mu V^\sigma \equiv \partial_\mu V^\sigma + \Gamma^\sigma_{\mu\tau}V^\tau \tag{C.10}$$

So

$$R_{ij}{}^k{}_l = R_{ij}{}^k{}_l{}^{[\mathrm{base}]} \tag{C.11}$$

$$R_{iI}{}^J{}_j = \frac{1}{2}(M^{-1}\nabla^{[\mathcal{B}]}{}_i\nabla^{[\mathcal{B}]}{}_j M)^J{}_I - \frac{1}{4}(M^{-1}M_{,i}M^{-1}M_{,j})^J{}_I \tag{C.12}$$

$$R_{iI}{}^j{}_J = -\frac{1}{2}\nabla^{[\mathcal{B}]}{}_i\nabla^{[\mathcal{B}]^j}M_{IJ} + \frac{1}{4}((\nabla^{[\mathcal{B}]^j}M)M^{-1}(\nabla^{[\mathcal{B}]}{}_i M))_{IJ} \tag{C.13}$$

$$R_{IJ}{}^i{}_j = \frac{1}{4}((\nabla^{[\mathcal{B}]^i}M)(M^{-1}\nabla^{[\mathcal{B}]}{}_j M))_{JI} - (I \leftrightarrow J) \tag{C.14}$$

$$R_{ij}{}^I{}_J = \frac{1}{4}(M^{-1}(\nabla^{[\mathcal{B}]}{}_j M)M^{-1}(\nabla^{[\mathcal{B}]}{}_i M))^I{}_J - (i \leftrightarrow j) \tag{C.15}$$

$$R_{IJ}{}^K{}_L = \frac{1}{4}(\nabla^{[\mathcal{B}]i} M_{IL})(M^{-1}\nabla^{[\mathcal{B}]}{}_i M)^K{}_J - (I \leftrightarrow J) \tag{C.16}$$

$$R_{iJ}{}^k{}_l = R_{iI}{}^J{}_K = R_{ij}{}^k{}_I = R_{ij}{}^I{}_k = R_{IJ}{}^K{}_i = R_{IJ}{}^i{}_K = 0\,. \tag{C.17}$$

D. Derivation of the BPS Equations for the two-dimensional case

As our basis of four-dimensional gamma matrices we take

$$\Gamma^{a=1} = \sigma^1 \otimes \sigma^1 \qquad \Gamma^{a=2} = \sigma^2 \otimes \sigma^1 \tag{D.1}$$

$$\Gamma^{A=1} = \sigma^3 \otimes \sigma^1 \qquad \Gamma^{A=2} = 1 \otimes \sigma^2 \tag{D.2}$$

$$\Gamma \equiv -\Gamma_{[4]} \equiv -\frac{1}{24}\epsilon^{\underline{\mathbf{ABCD}}}\Gamma^{\underline{\mathbf{A}}}\Gamma^{\underline{\mathbf{B}}}\Gamma^{\underline{\mathbf{C}}}\Gamma^{\underline{\mathbf{D}}} = 1 \otimes \sigma^3 \tag{D.3}$$

Projecting onto spinors ψ of positive chirality $\Gamma\psi = \psi$, we define

$$\Gamma^{\underline{\mathbf{A}}B} \equiv -\frac{i}{2}[\Gamma^{\underline{\mathbf{A}}}, \Gamma^{\underline{\mathbf{B}}}] \tag{D.4}$$

which, in the basis we have chosen, gives

$$\Gamma^{ab} = \epsilon^{ab}\sigma^3 \qquad\qquad \Gamma^{AB} = \epsilon^{AB}\sigma^3 \tag{D.5}$$

$$\Gamma^{a=\pm, A=\pm} = \pm 2i\sigma^{\pm} \tag{D.6}$$

$$\Gamma^{a=\pm, A=\mp} = 0 \tag{D.7}$$

where

$$V^{\pm} \equiv V^1 \pm iV^2 \tag{D.8}$$

and the Pauli matrices σ^p are the usual ones satisfying $\sigma^p\sigma^q = \delta^{pq} + i\epsilon^{pqr}\sigma^r$.

D.1 Killing spinor equations along the fiber

A subalgebra of $\mathrm{SO}(4) = \mathrm{SU}(2)\times\mathrm{SU}(2)$ can only preserve one spinor if it preserves two, and both must be of the same chirality. So the condition we want is actually that the covariant derivative actually annihilate *two* linearly independent spinors of the same chirality, χ_4. Given our ansatz, the condition $\nabla_I\psi = 0$ does not involve the partial derivative ∂_I so it only involves $\Omega_I{}^{\underline{\mathbf{AB}}}$ which means it only involves $\Omega_I{}^{aA}$ because the other components vanish. Using the identity

$$\Gamma^{\underline{\mathbf{A}}B} = \epsilon^{\underline{\mathbf{A}}BCD}\Gamma_{[4]}\Gamma^{\underline{\mathbf{C}}D}\,,$$

the condition is that the matrix $\Omega_I{}^{aA}\Gamma^{aA}$ be (anti-) self-dual,

$$0 = \Omega_I{}^{aA} + \chi_4\epsilon^{AB}\epsilon^{ab}\Omega_I{}^{bB} \equiv Q_I^{aA}\,. \tag{D.9}$$

Contracting this equation into f^{AI}, we learn that

$$0 = \frac{2}{V}\left(\frac{1}{2}e^{ia}\partial_i V + \chi_6\epsilon^{ab}e^{ib}b_i\right)$$

which is equivalent to the Cauchy-Riemann equations

$$0 = \bar{\partial}\rho\,.$$

Contracting (D.9) into $\epsilon^{AC} f^{IC}$ leads to the same equation. The only remaining independent part of (D.9) can be extracted by contracting it with $f^{BI} V^A V^B$, where $V^A = (1, i)$. This gives

$$0 = Q_I^{aA} f^{BI} V^A V^B = \frac{1}{2} H^I H^J M_{IJ,i} \left(e^{ia} - i\chi_4 \epsilon^{ab} e^{ib} \right) \tag{D.10}$$

where $H^I \equiv f^{A=1\ I} + i f^{A=2\ I}$. Eq. (D.10) is equivalent to

$$0 = \epsilon_{IJ} H_J \left(\partial_1 - i\chi_4 \partial_2 \right) H_I \tag{D.11}$$

If we choose $\chi_4 = 1$, this is $0 = \epsilon_{IJ} H^I \bar{\partial} H^J$. Since H_I can never be zero, is equivalent to

$$\partial_{\bar{z}} H_I = \lambda H_I \tag{D.12}$$

for some function $\lambda(z, \bar{z})$. We are working locally so we can always set λ equal to $\partial_{\bar{z}} L$ for some function $L(z, \bar{z})$, and so the equation above is solved by

$$H_I = \exp\left\{ L(z, \bar{z}) \right\} G_I(z) \tag{D.13}$$

for some function L and holomorphic functions G_I. (Remember that globally neither L nor G_I need be single valued!)

We can in fact determine L by using the fact that $\det f = V$. We have

$$V = \det f = \frac{1}{2} \epsilon_{IJ} \epsilon^{AB} f_I^A f_J^B = \frac{1}{2} \epsilon_{IJ} (f_I^1 f_J^2 - f_I^2 f_I^1) = \frac{i}{2} \exp\left\{ 2\mathrm{Re}(L) \right\} \epsilon_{IJ} G_I G_J^* \tag{D.14}$$

and so we end up with

$$H_I(z, \bar{z}) = \sqrt{V} \cdot \left[-\frac{i}{2} \epsilon_{JK} G_J(z) G_K^*(\bar{z}) \right]^{-\frac{1}{2}} \exp\left\{ iK(z, \bar{z}) \right\} G_I(z) \tag{D.15}$$

$$f_I^{A=1} = \mathrm{Re}(H_I) \qquad f_I^{A=2} = \mathrm{Im}(H_I) \tag{D.16}$$

as the most general solution (locally) to the condition that $\nabla_I \psi = 0$ for spinors of positive chirality.

Next, notice that changing the quantity G_I by overall multiplication by a holomorphic function $G_I \to f(z) \cdot G_I$ changes H_I only by a phase $K \to K + \arg(f(z))$. So only the ratio $\tau(z) \equiv G_1/G_2 = H_1/H_2$ is physical, and it satisfies the equation $\partial_{\bar{z}} \tau = 0$. It is not difficult to check that τ is indeed the complex structure τ defined earlier.

D.2 The other half of the Killing spinor equations

Having exhausted the content of the equation $\nabla_I \psi = 0$, we turn to $\nabla_i \psi = 0$. First we work out the Christoffel symbols, in conformal gauge:

$$G_{ij} = g_{ij} = \exp\left\{ 2\varphi \right\} \delta_{ij} \qquad e_a^i = \delta^{ia} \exp\left\{ \varphi \right\} . \tag{D.17}$$

Then

$$\Gamma_{ij}^k = (\delta_{ik} \varphi_{,j} + \delta_{jk} \varphi_{,i} - \delta_{ij} \varphi_{,k}) \tag{D.18}$$

$${\omega_i}^{ab} = \varphi_{,j} (\delta^{ia} \delta^{jb} - \delta^{ib} \delta^{ja}) = \varphi_{,j} \epsilon_{ij} \epsilon_{ab} \tag{D.19}$$

$${\omega_{i=1}}^{ab} + i {\omega_{i=2}}^{ab} = -i \epsilon^{ab} \partial_{\bar{z}} \varphi \tag{D.20}$$

$${\omega_{i=1}}^{ab} - i {\omega_{i=2}}^{ab} = +i \epsilon^{ab} \partial_z \varphi \tag{D.21}$$

$${\omega_i}^{AB} = -\frac{1}{4V} \epsilon^{AB} \epsilon_{IJ} (H_I H_{J,i}^* + H_I^* H_{J,i}) . \tag{D.22}$$

Parametrizing H_I as

$$[\,H_1 H_2\,] = \exp\{iK\} \cdot \left(\frac{V}{\tau_2}\right)^{1/2} [\,\tau 1\,]\,, \tag{D.23}$$

we get

$$\omega_i{}^{AB} = -\frac{1}{4\tau_2}\epsilon^{AB}(4\tau_2 K_{,i} - 2\tau_{1,i}) \tag{D.24}$$

so, using equation for τ derived in the previous subsection, we have

$$\partial\tau_1 \;=\; -i\chi_4\partial\tau_2 \qquad \bar{\partial}\tau_1 \;=\; +i\chi_4\bar{\partial}\tau_2 \tag{D.25}$$

$$\omega_z{}^{AB} \;=\; \epsilon^{AB}\partial_z\left(-K - \frac{i}{2}\chi_4 \ln\tau_2\right) \tag{D.26}$$

$$\omega_{\bar{z}}{}^{AB} \;=\; \epsilon^{AB}\partial_{\bar{z}}\left(-K + \frac{i}{2}\chi_4 \ln\tau_2\right). \tag{D.27}$$

Defining the generalized spin connection matrix, $\hat{\Omega}_\mu \equiv \Omega_\mu{}^{\underline{\mathbf{AB}}}\Gamma^{\underline{\mathbf{AB}}}$, (with $\Omega_\mu{}^{\underline{\mathbf{AB}}}$ as defined in (2.22)) which enters into the action of the covariant derivative on spinors as

$$\tilde{\nabla}_\mu\psi_\alpha \equiv \partial_\mu\psi_\alpha - \frac{i}{4}(\hat{\Omega}_\mu)_{\alpha\beta}\psi_\beta, \tag{D.28}$$

we have

$$\hat{\Omega}_{\bar{z}} \equiv \hat{\Omega}_{i=1} + i\hat{\Omega}_{i=2} = 2\left(-i\partial_{\bar{z}}\varphi - \frac{1}{2V}\epsilon_{IJ}H_I^*\partial_{\bar{z}}H_J + \frac{\bar{\partial}b}{V}\right)\cdot\sigma^3\,; \tag{D.29}$$

$\hat{\Omega}_z$ is the complex conjugate of this expression.

The condition for the existence of a covariantly constant spinor is then

$$0 = \left[\partial_z - \frac{i}{4}\hat{\Omega}_z, \partial_{\bar{z}} - \frac{i}{4}\hat{\Omega}_{\bar{z}}\right]. \tag{D.30}$$

Using the equation found by imposing the vanishing of the dilatino variation,

$$\frac{\bar{\partial}b}{V} = i\chi_6\bar{\partial}\Phi\,,$$

this is

$$0 \;=\; -4i\partial_z\partial_{\bar{z}}\varphi - \frac{1}{V}\epsilon_{IJ}\partial_z H_I^*\partial_{\bar{z}}H_J - \frac{1}{V}\epsilon_{IJ}H_I^*\partial_z\partial_{\bar{z}}H_J - 4i\chi_6\partial\bar{\partial}\Phi \tag{D.31}$$

$$\;=\; -4i\partial_z\partial_{\bar{z}}(\varphi + \chi_6\Phi) - \frac{1}{V}\partial_z(\epsilon_{IJ}H_I^*\partial_{\bar{z}}H_J)\,. \tag{D.32}$$

Using

$$\epsilon_{IJ}H_I^*\partial_{\bar{z}}H_J = -2iV\partial_{\bar{z}}R \tag{D.33}$$

where

$$R \equiv iK - \frac{1}{2}\ln[-\frac{i}{2}\epsilon_{IJ}G_I G_J^*]\,.$$

Rewriting $\bar{\partial}\Phi = -\frac{1}{2}\bar{\partial}\ln\rho_2$, and noting that $\partial\bar{\partial}R = \frac{1}{2}\partial\bar{\partial}\ln\tau_2$, the condition (D.30) becomes

$$0 = \partial_z\partial_{\bar{z}}\left(\varphi - \frac{1}{2}\ln\tau_2 - \frac{1}{2}\ln\rho_2\right). \tag{D.34}$$

E. Vector modes in the base

In this appendix, we consider solutions in which the off-block-diagonal components of the metric and B-field are nonzero — i.e. in which there are 8D vectors turned on. This more general ansatz is written in equations (2.11) and (2.12). The Christoffel symbols and (generalized) spin connection become

$$
\begin{aligned}
\Gamma_{ij;k} &= \gamma_{ij;k} + \frac{1}{2}M_{IJ,j}A_i^I A_k^J + \frac{1}{2}M_{IJ,i}A_j^I A_k^J - \frac{1}{2}M_{IJ,k}A_i^I A_j^J + \\
&\quad + \frac{1}{2}M_{IJ}(A^I_{k,i}A^J_j + A^I_j A^J_{k,i} + A^I_{k,j}A^J_i + A^I_j A^J_{k,i} - A^I_{i,k}A^J_j - A^I_i A^J_{j,k}) && \text{(E.1)}\\
\Gamma_{ij;K} &= -\frac{1}{2}M_{KL}(A^L_{i,j} + A^L_{j,i}) - \frac{1}{2}(A^L_i M_{KL,j} + A^L_j M_{KL,i}) && \text{(E.2)}\\
\Gamma_{iJ;k} &= -\frac{1}{2}M_{JL}F^L_{ik} - \frac{1}{2}A^L_k M_{JL,i} + \frac{1}{2}A^L_i M_{JL,k} && \text{(E.3)}\\
\Gamma_{iJ;K} &= \frac{1}{2}M_{JK,i} && \text{(E.4)}\\
\Gamma_{IJ;k} &= -\frac{1}{2}M_{IJ,k} && \text{(E.5)}\\
\Gamma_{IJ;K} &= 0 && \text{(E.6)}\\
\Gamma^k_{ij} &= \frac{1}{2}M_{IJ}g^{kl}(A^J_j F^I{}_{il} + A^J_i F^I_{jl}) - \frac{1}{2}g^{kl}M_{IJ,l}A^I_i A^J_j && \text{(E.7)}\\
\Gamma^I_{jk} &= \frac{1}{2}A^{Il}M_{JK}(A^J_j F^K_{kl} + A^J_k F^K_{jl}) - \frac{1}{2}(\nabla^{[B]}_i A^I_j + \nabla^{[B]}_j A^I_i) - \\
&\quad - \frac{1}{2}M^{IJ}(M_{JK,k}A^K_j + M_{JK,j}A^K_k) - \frac{1}{2}A^{Ii}A^J_j A^L_k M_{JL,i} && \text{(E.8)}\\
\Gamma^i_{jK} &= -\frac{1}{2}M_{IK}g^{il}F^I_{jl} + \frac{1}{2}g^{il}A^I_j M_{IK,l} && \text{(E.9)}\\
\Gamma^I_{jK} &= -\frac{1}{2}M_{KL}F^L_{ji}A^{Ii} + \frac{1}{2}A^L_j A^{Ii}M_{KL,i} + \frac{1}{2}M^{IJ}M_{JK,j} && \text{(E.10)}\\
\Gamma^i_{JK} &= -\frac{1}{2}g^{ij}M_{JK,j} && \text{(E.11)}\\
\Gamma^I_{JK} &= -\frac{1}{2}A^{Ii}M_{JK,i} && \text{(E.12)}\\
\omega_i{}^{ab} &= \omega_i{}^{ab[\mathrm{B}]} - \frac{1}{2}M_{IJ}e^{aj}e^{bk}A^I_i F^J_{jk} && \text{(E.13)}\\
\omega_i{}^{Aa} &= \frac{1}{2}e^{aj}f^{AI}(M_{IJ}F^J_{ij} - A^J_i M_{IJ,j}) && \text{(E.14)}\\
\omega_i{}^{AB} &= \frac{1}{2}f^{AI}f^B_{I,i} - \frac{1}{2}f^{BI}f^A_{I,i} && \text{(E.15)}\\
\omega_I{}^{ab} &= +\frac{1}{2}M_{IJ}e^{ai}e^{bj}F^J_{ij} && \text{(E.16)}\\
\omega_I{}^{Aa} &= +\frac{1}{2}f^{AJ}e^{ai}M_{IJ,i} && \text{(E.17)}\\
\omega_I{}^{AB} &= 0 && \text{(E.18)}\\
\Omega_i{}^{ab} &= \omega_i{}^{ab[\mathrm{B}]} - \frac{1}{2}M_{IJ}e^{aj}e^{bk}A^I_i F^J_{jk} && \text{(E.19)}\\
\Omega_i{}^{Aa} &= \frac{1}{2}e^{aj}f^{AI}(M_{IJ}F^J_{ij} - A^J_i M_{IJ,j} - \chi_{10}H_{Iij}) && \text{(E.20)}
\end{aligned}
$$

JHEP01(2004)024

$$\Omega_i{}^{AB} = \frac{1}{2}f^{AI}f^B_{I,i} - \frac{1}{2}f^{BI}f^A_{I,i} + \chi_{10}\frac{\partial_i b}{V}\epsilon^{AB} \tag{E.21}$$

$$\Omega_I{}^{ab} = e^{ai}e^{bj}(\frac{1}{2}M_{IJ}F^J_{ij} + \chi_{10}H_{Iij}) \tag{E.22}$$

$$\Omega_I{}^{Aa} = f^{AJ}e^{ai}(\frac{1}{2}M_{IJ,i} + \chi_{10}\epsilon_{IJ}\partial_i b) \tag{E.23}$$

$$\Omega_I{}^{AB} = 0\,. \tag{E.24}$$

In the general case, where the one-forms in the base are nonzero, the BPS equations are satisfied if their field strengths vanish, i.e.

$$H_{Iij} = F^I_{ij} = 0\,. \tag{E.25}$$

This follows from working out the BPS equations explicitly, but for brevity we omit these calculations and point out that this fact can be derived on general grounds. A spinor field transforms as a scalar under general coordinate transformations and also trivially under gauge transformations of the B-field. Since the gauge transformations of the one-forms in the base are inherited from these local symmetries, it follows that both they and the (vielbein-contracted) covariant derivatives $\nabla_a \equiv E^\mu_a\nabla_\mu, \nabla_A \equiv E^\mu_A\nabla_\mu$ are gauge-neutral. It follows that vectors in the base cannot appear in the BPS equations except through their field strengths. With the field strengths set to zero, then, the BPS equations reduce to the ones we derived with the off-diagonal components of the metric and B-field set to zero.

From this analysis we conclude that the BPS equations we found above are unmodified by turning on KK vectors or off-diagonal modes of the NS B-field if these have vanishing field strength. As such, the analysis presented in section 4 counting zeromodes of these fields is appropriate.

We also find that $0 = \tilde{\nabla}_I\psi$ is solved by

$$0 = \frac{1}{2}M_{IJ}F^J_{ij} + H_{Iij}\,. \tag{E.26}$$

The BPS equations also imply the hermitean Yang-Mills equations $J^k{}_iJ^l{}_jF^I_{kl} - F^I_{ij} = g^{ik}J^j{}_kF^I_{ij} = 0$ for the complementary linear combination of the fluxes. For the case of a four-dimensional base, these solutions dualize to solutions of type IIB string theory with dilaton gradients (a.k.a. 'F theory') and RR fluxes, of the kind discussed in [34]. It seems likely that by combining these two complementary moduli-fixing mechanisms, F theory on nongeometric compactifications with RR flux could fix moduli completely, or at least generate solutions isolated from any type of large-volume point or dual thereof.

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JHEP01(2004)024